Lightcone gauge in covariant SFT

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EUROPEAN UNION
European Structural and Investment Funds
Operational Programme Research,
Development and Education

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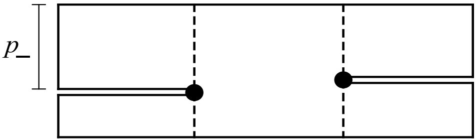
Lightcone SFT: Mandelstam, Kaku/Kikkawa,... (early 1970s)

String field: $\Psi_{\mathrm{lc}} \in C^\infty(\mathbb{R}^{1,1}) \otimes \mathcal{H}_{X^i} = \mathrm{Span}(\alpha^i_{-n}...|k_\mu\rangle)$

$c = 24$

Gauge invariance: None.

Interactions: Mandelstam diagrams.

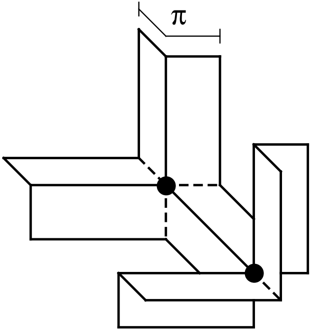


Covariant SFT: Witten, Zwiebach,... (late 1980s)

String field: $\Psi_{\text{cov}} \in \underbrace{\mathcal{H}_{X^\mu}}_{c=26} \otimes \underbrace{\mathcal{H}_{bc}}_{c=-26} = \text{Span}(\alpha^{\mu}_{-n}...b_{-m}...c_{-l}|k_\mu\rangle)$

Gauge invariance: $\delta\Psi_{\text{cov}} = Q\Lambda + \text{higher orders in } \Psi_{\text{cov}}$

Interactions: e.g. Witten's theory.



Elementary question: How are these SFTs related?

The rough answer is basically what you would expect, namely:

Lightcone SFT is covariant SFT fixed to lightcone gauge.

Less obviously, the lightcone gauge condition turns out to be:

$$\left(b_0 + ip_- \oint \frac{d\xi}{2\pi i}\frac{b(\xi)}{\partial X^+(\xi)}\right)\Psi_{\text{cov}} = 0$$

How this works:

The covariant string field can be decomposed into transverse and longitudinal parts:

$$\Psi_{\text{cov}} = \text{transverse} + \text{longitudinal}$$

- transverse is isomorphic to the standard lightcone string field Ψ_{lc}.
- longitudinal is subject to purely algebraic equations of motion when the lightcone gauge condition is satisfied. Therefore it can be integrated out.

The result is an action for Ψ_{lc} without gauge symmetry. Therefore it can be called "lightcone SFT."

However, lightcone SFT (as conventionally understood) is characterized not only by Ψ_{lc}, but by the fact that interactions take place through Mandelstam diagrams.

It is far from clear that fixing lightcone gauge in a typical covariant SFT results in a conventional lightcone SFT.

However:

Kaku-Kikkawa lightcone SFT = Kugo-Zwiebach SFT in lightcone gauge

(T.E. and H. Matsunaga, 2020)

The Kugo-Zwiebach SFT is a covariant SFT whose interactions are defined by lightcone-style vertices.

Lightcone vertices are preserved through the process of fixing lightcone gauge and integrating out longitudinal states.

→ transfer invariance.

However, the Kugo-Zwiebach theory is not quite a covariant SFT, since lightcone interactions are not Lorentz invariant.

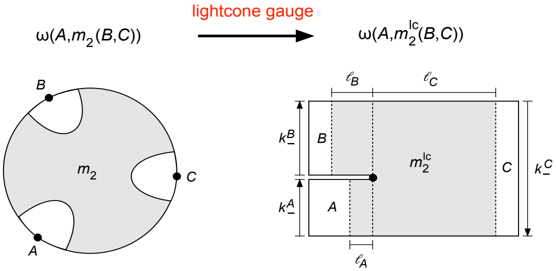
Moreover, while transfer invariance is not trivial, perhaps it is not surprising that we get conventional lightcone interactions from a covariant SFT which already has such interactions.

This leads to the question:

What kind of "lightcone SFT" results from fixing lightcone gauge in a typical covariant SFT?

This is the question we address in this talk.

Cubic vertex



In lightcone gauge, the cubic vertex reduces to a cubic lightcone vertex attached to stubs.

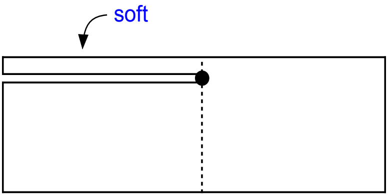
Remarkably, regardless of the geometry of the covariant cubic vertex, once we fix lightcone gauge a Mandelstam diagram appears.

The only information about the covariant vertex that survives in lightcone gauge is the local dilatation at the punctures.

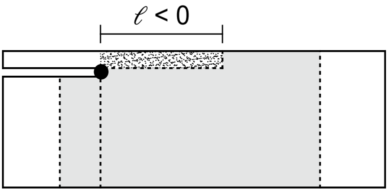
The lengths of the stubs attached to the cubic lightcone vertex are fixed so that the local dilatation at the punctures is the same in the covariant theory as it is after fixing lightcone gauge.

In other words, for primary states the vertex is the same before and after fixing lightcone gauge.

However, there is a strange difficulty. If one string is “soft,” the local dilatation at the corresponding puncture is very small.



Since the local dilatation at the puncture of a covariant vertex is fixed, the stub attached to this state needs to be shortened. Eventually, its length will need to be negative:



This is not a normalizable string vertex.

Therefore lightcone gauge in covariant SFT is generically singular.

This is called the soft string problem of lightcone gauge. It continues to be an obstacle to formulating a precise relation between covariant and lightcone SFTs.

In the following we will assume that amplitudes are evaluated in a kinematic region where lightcone gauge is well-defined.

Next: What happens to higher order interactions in lightcone gauge?

Details of lightcone gauge

Important operators:

$$b_{\text{DDF}} = -ip_- \oint \frac{d\xi}{2\pi i} \frac{b(\xi)}{\partial X^+(\xi)}$$

$$L_{\text{DDF}} = 2p_+p_- - ip_- \oint \frac{d\xi}{2\pi i} \frac{T^{X^i}(\xi) - 2\{X^+, \xi\}}{\partial X^+(\xi)} = [Q, b_{\text{DDF}}]$$

Note that b_{DDF} and L_{DDF} are conformally invariant operators

L_{DDF} is the level counting operator for DDF/transverse excitations in the string field.

If Ψ_{cov} is the covariant string field, lightcone gauge is:

$$\big(b_0 - b_{\text{DDF}}\big)\Psi_{\text{cov}} = 0$$

- If $L_0 = L_{\text{DDF}}$, all excitations in the string field are transverse.
- If $L_0 > L_{\text{DDF}}$ some excitations of the string field come from unphysical ghost and longitudinal oscillator excitations.

Therefore we have decomposition

$$\Psi_{\text{cov}} = \overbrace{S\Psi_{\text{lc}}}^{L_0=L_{\text{DDF}}} + \overbrace{S\Psi_{\text{unphysical}}}^{L_0>L_{\text{DDF}}}$$

turns transverse oscillators
into DDF operators

Specifically:

$$S\alpha^i_{-n} = e^{\frac{in}{2p_-}x^+} \overbrace{\left(i\sqrt{2} \oint \frac{d\xi}{2\pi i} e^{-\frac{in}{p_-}X^+(\xi)} \partial X^i(\xi) \right)}^{\text{DDF operator}} S$$

conformally invariant

zero mode prefactor

The propagator in lightcone gauge breaks into transverse and longitudinal parts

$$\Delta_{\mathrm{lc}} = \frac{b_0}{L_0}\delta(L_0 - L_{\mathrm{DDF}}) + \underbrace{\frac{b_0 - b_{\mathrm{DDF}}}{L_0 - L_{\mathrm{DDF}}}}_{\Delta_{\mathrm{long}}}$$

The longitudinal propagator does not have any poles, so the longitudinal states can be integrated out.

If the original action is

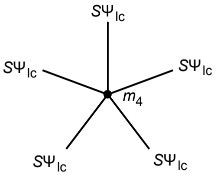
$$S = \frac{1}{2}\omega(\Psi_{\mathrm{cov}}, Q\Psi_{\mathrm{cov}}) + \frac{1}{3}\omega(\Psi_{\mathrm{cov}}, m_2(\Psi_{\mathrm{cov}}, \Psi_{\mathrm{cov}})) + \frac{1}{4}\omega(\Psi_{\mathrm{cov}}, m_3(\Psi_{\mathrm{cov}}, \Psi_{\mathrm{cov}}, \Psi_{\mathrm{cov}})) + \ldots$$

the resulting gauge fixed action is

$$S_{\mathrm{lc}} = \frac{1}{2}\omega(\Psi_{\mathrm{lc}}, c_0 L_0 \Psi_{\mathrm{lc}}) + \frac{1}{3}\omega(\Psi_{\mathrm{lc}}, m_2^{\mathrm{lc}}(\Psi_{\mathrm{lc}}, \Psi_{\mathrm{lc}})) + \frac{1}{4}\omega(\Psi_{\mathrm{lc}}, m_3^{\mathrm{lc}}(\Psi_{\mathrm{lc}}, \Psi_{\mathrm{lc}}, \Psi_{\mathrm{lc}})) + \ldots$$

The products $m_2^{\mathrm{lc}}, m_3^{\mathrm{lc}}, \ldots$ are given by a Feynman graph expansion, where the vertices are those of the original covariant SFT and the internal lines represent longitudinal propagators.

$\omega(\Psi_{lc}, m_4^{lc}(\Psi_{lc},\Psi_{lc},\Psi_{lc},\Psi_{lc})) =$



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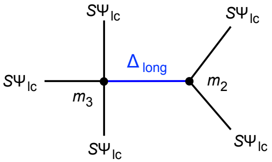

$+$ cyclic

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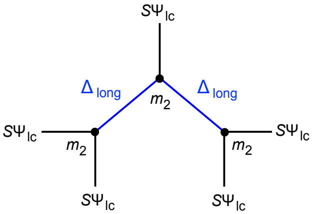


$+$ cyclic

Vertices without longitudinal propagators

For open strings, the n-point vertex in a covariant SFT is given by integrating surface states $\langle\Sigma_n|$ over some portion of the moduli space of disks with boundary punctures.

The surface states contribute to the lightcone gauge action as

$$\langle\Sigma_n|SA_1 \otimes ...SA_n$$

where SA_i are transverse string states.

Point 1:

Since S maps transverse oscillators into DDF operators, the local coordinate maps which define $\langle\Sigma_n|$ only effect the result through the zero mode prefactor.

$$\begin{aligned} f\circ\left[e^{\frac{in}{2p_-}x^+}\left(i\sqrt{2}\oint_0\frac{d\xi}{2\pi i}e^{-\frac{in}{p_-}X^+(\xi)}\partial X^i(\xi)\right)\right] \\ = e^{\frac{in}{2p_-}f\circ x^+}\left(i\sqrt{2}\oint_{f(0)}\frac{du}{2\pi i}e^{-\frac{in}{p_-}X^+(u)}\partial X^i(u)\right)\end{aligned}$$

Therefore, much of the geometrical information which defines the covariant n-string vertex is lost when we fix lightcone gauge.

Point 2:

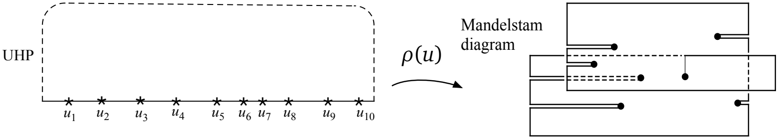
Let $F[X^+(u)]$ be an operator on the upper half plane which depends on chiral free boson $X^+(u)$ but not $X^-(u)$. Then

$$\left\langle F[X^+(u)] e^{ik^1\cdot X(u_1,u_1)} \ldots e^{ik^n\cdot X(u_n,u_n)} \right\rangle_{\text{UHP}}$$
$$= \left\langle F\left[-\frac{i}{2}\rho(u)\right] e^{ik^1\cdot X(u_1,u_1)} \ldots e^{ik^n\cdot X(u_n,u_n)} \right\rangle_{\text{UHP}}$$

where

$$\rho(u) = \sum_{i=1}^{n} 2k_-^i \ln(u - u_i)$$

is the Mandelstam mapping.



This is called the replacement formula.

It says that in the context of this correlator, the coordinate *rho* on a Mandelstam diagram is proportional to the plus component of the string embedding coordinate.

It is as though the string worldsheet has been quantized in lightcone gauge.

Using the replacement formula, the DDF operators evaluate to

$$i\sqrt{2}\oint \frac{du}{2\pi i} e^{-\frac{in}{p_-} X^+(u)} \partial X^i(u) = i\sqrt{2}\oint \frac{du}{2\pi i} e^{-\frac{n}{2p_-}\rho(u)} \partial X^i(u)$$

Remarkably, this is the same as a conformal transformation of the transverse oscillator:

$$\rho^{-1} \circ (2p_- \ln) \circ \alpha^i_{-n}$$

The geometry of the surface state is replaced by the geometry of a Mandelstam diagram!

Precise statement:

$$\langle\Sigma_n|SA_1 \otimes \ldots SA_n = \langle\Sigma_n^{\mathrm{lc}}|\left(e^{-\lambda_1 L_0} \otimes \ldots \otimes e^{-\lambda_n L_0}\right) A_1 \otimes \ldots \otimes A_n$$

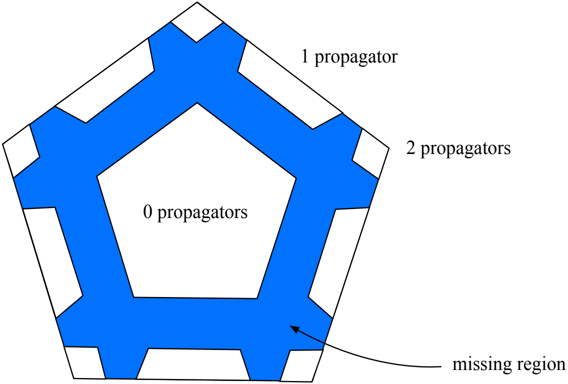
where

- $\langle\Sigma_n^{\mathrm{lc}}|$ is the canonical surface state associated to the Mandelstam diagram at the same point in moduli space as $\langle\Sigma_n|$,
- Stub length parameters $\lambda_1, ..., \lambda_n$ ensure that the local dilitation at the punctures agree of both sides.

The covariant vertex between transverse states reduces to a collection of Mandelstam diagrams representing the same portion of moduli space.

But since the local coordinates are changed, the Feynman diagrams defined by the covariant vertices alone do not cover the moduli spaces of Riemann surfaces in lightcone gauge.

For example, 5-point amplitudes:



To get the missing region, we must apparently account for the propogation of longitudinal states.

Dealing with longitudinal propagators raises two questions:

1. A strip of longitudinal worldsheet has nonvanishing central charge. Therefore the worldsheet partition function is nontrivial.
2. How does a strip of longitudinal worldsheet move in moduli space?

Partition functions on Mandelstam diagrams

Kugo-Zwiebach phenomenon:

Transverse Siegel gauge amplitudes are equal to lightcone gauge amplitudes in the Kugo-Zwiebach theory.

It does not matter whether we use

$$\frac{b_0}{L_0} \quad \text{or} \quad \frac{b_0}{L_0}\delta(L_0 - L_{\mathrm{DDF}}) + \Delta_{\mathrm{long}}$$

for the propagator!

Due to transfer invariance of lightcone vertices, we know that we can drop Δ_{long} from the lightcone gauge propagator.

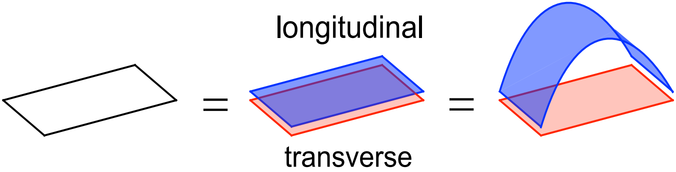
Therefore it does not matter whether the propagator is

$$\frac{b_0}{L_0} \quad \text{or} \quad \frac{b_0}{L_0}\delta(L_0 - L_{\text{DDF}}) = \frac{b_0}{L_0}e^{-\infty(L_0 - L_{\text{DDF}})}$$

The latter suggests that transverse Siegel gauge amplitudes in the Kugo-Zwiebach theory do not contain longitudinal intermediate states.

If this is the case, we should be able to adjust the length of a propagator strip seperately in the longitudinal sector without changing the result.

$$e^{-sL_0} = e^{-sL_{\mathrm{DDF}}} e^{-s(L_0 - L_{\mathrm{DDF}})} = e^{-sL_{\mathrm{DDF}}} e^{-s'(L_0 - L_{\mathrm{DDF}})}$$



In this way, the transverse propagator strips of lightcone SFT can be accompanied with longitudinal propagator strips, obtaining correlation function on in worldsheet with vanishing central charge.

This gives a way to handle the conformal anomaly and partition functions on Mandelstam diagrams in lightcone SFT.

Vertices with longitudinal propagators

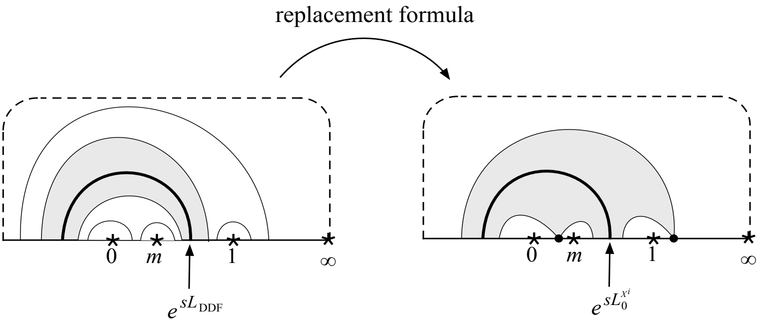
The longitudinal propagator takes the form:

$$\Delta_{\text{long}} = (b_0 - b_{\text{DDF}}) \int_0^\infty ds e^{-s(L_0 - L_{\text{DDF}})}$$

The trick to is to let the e^{-sL_0} factor generate surface which defines the worldsheet correlation function.

The factor $e^{sL_{\text{DDF}}}$ is then treated separately as an operator insertion in the correlation function.

For the 4-point amplitude, this leads to a correlation function of the following form:



Applying the replacement formula, the local coordinates for the Feynman diagram of the covariant theory are replaced by local coordinates at the same point in moduli space in the lightcone theory.

The operator L_{DDF} is replaced by the Virasoro generator $L_0^{X^i}$ of the transverse CFT.

In this way the longitudinal propagator reduces to the integral

$$\int_0^\infty ds\, e^{-s_{\mathrm{lc}}(s) L_0 + s L_0^{X^i}}$$

where $s_{\mathrm{lc}}(s)$ is the Schwinger parameter in the lightcone diagram which corresponds to the same point in moduli space as the Schwinger parameter s in the covariant diagram.

Moreover, we can adjust the length of a strip of longitudinal worldsheet to replace

$$s_{\mathrm{lc}}(s) L_0 - s L_0^{X^i} \;\rightarrow\; (s_{\mathrm{lc}}(s) - s) L_0$$

Thus the longitudinal propagator gives

$$\int_0^\infty ds\, e^{-s_{\mathrm{lc}}(s) - s) L_0}$$

This integral precisely covers the missing region of moduli space.

Thank you!