

# The classical cosmological constant of open-closed string field theory



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Based on 2208.00410

**+work in progress**

with **Jakub Vosmera (ETH)**



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- D-branes arise as classical solutions of **open string field theory** (fixed closed string background)
- A change in the closed string background can be described through a classical solution in **closed string field theory**.
- D-branes in a changing closed string background should be described within **open-closed string field theory**.

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$$S^{(\mu)}[\Psi] = \frac{1}{2} \langle \Psi, Q\Psi \rangle + \frac{1}{3} \langle \Psi, \Psi^2 \rangle + \mu \langle \Psi, e \rangle$$

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- We showed that this approach can describe the fate of D-branes after a (continuous) change of the closed string background.

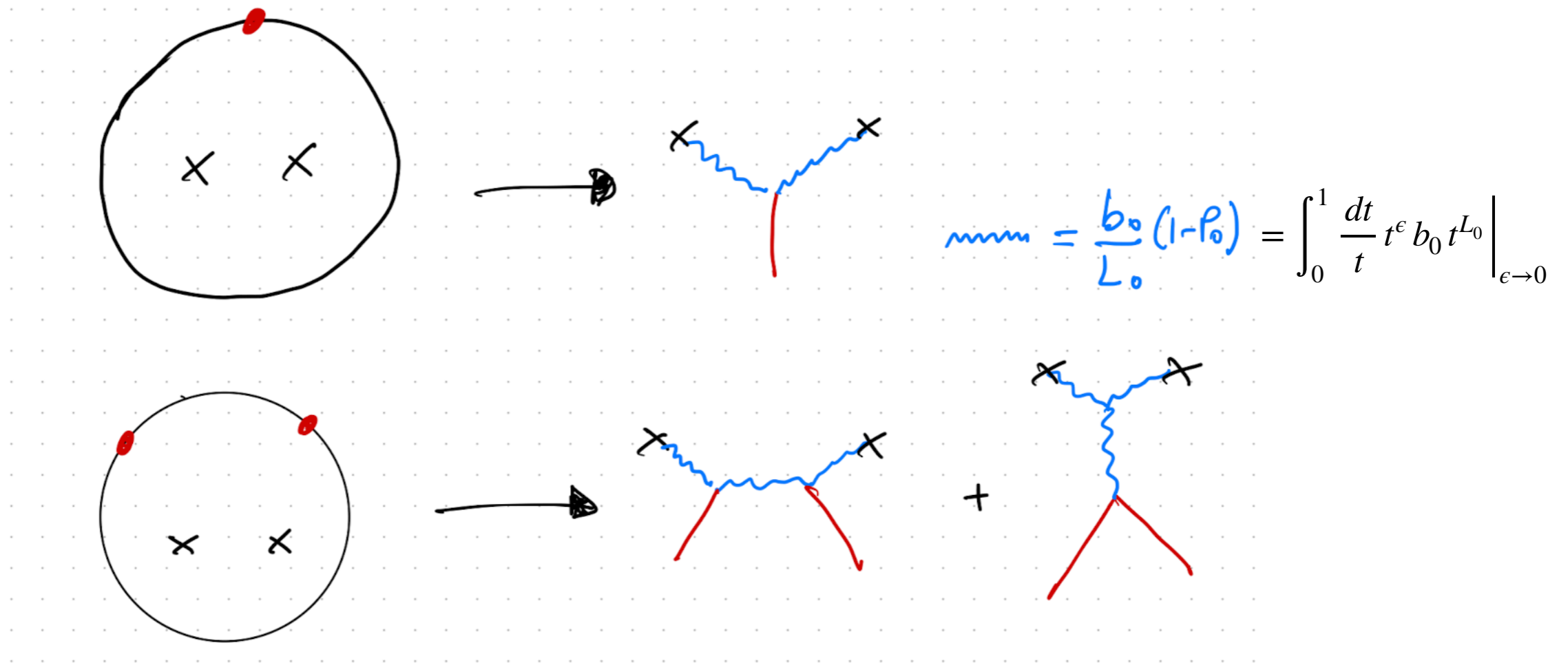


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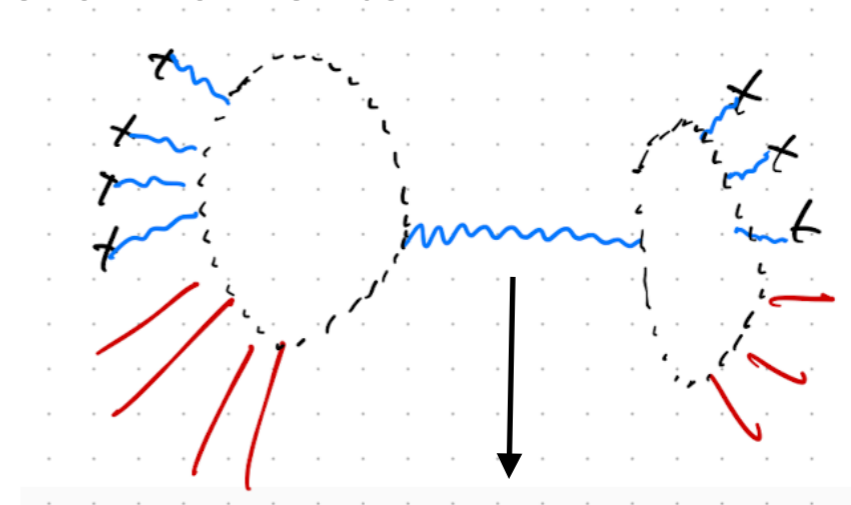
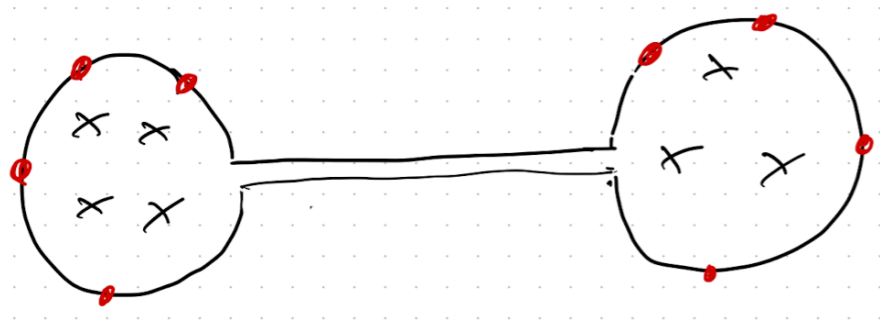
- This shift is encoded in disk amplitudes involving off-shell open strings and on-shell (deforming) closed strings. In principle all open-closed amplitudes are reproduced even at loop level, with the moduli space completely covered by the Schwinger parameters of the open string propagators. (*Zwiebach '91-'92*)



*etc...*

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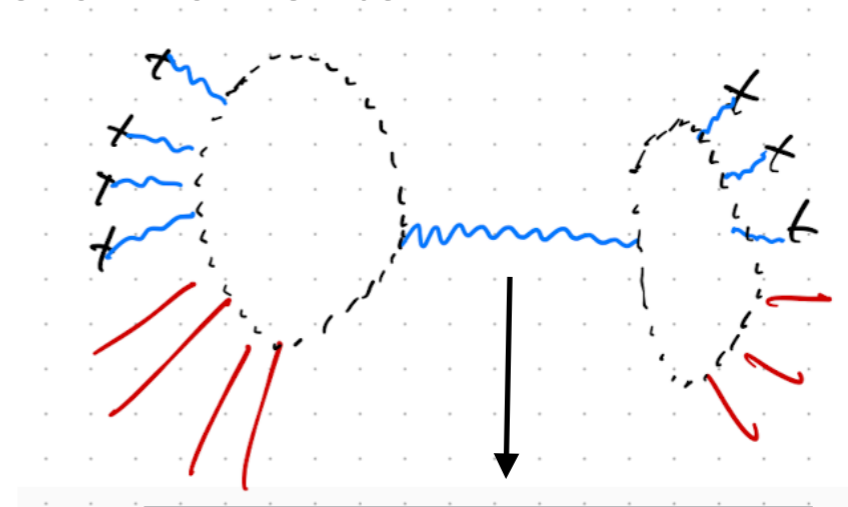
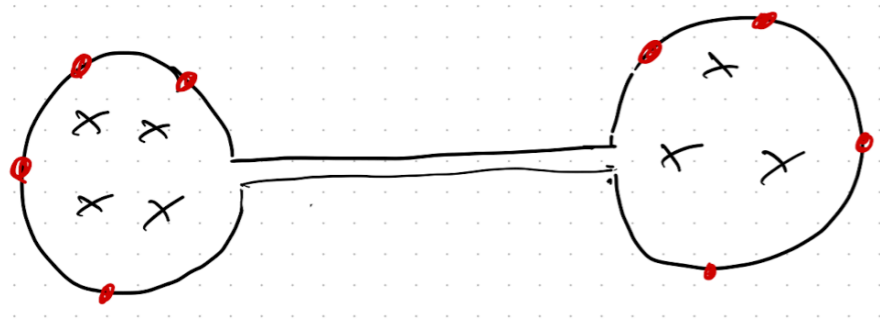
- Thanks to the open string propagators, these amplitudes are well-defined towards open string degeneration, even at zero momentum.



$$\frac{b_0}{L_0} = \frac{b_0}{L_0 + \epsilon} \Big|_{\epsilon \rightarrow 0} = \int_0^1 \frac{dt}{t} t^\epsilon b_0 t^{L_0} \Big|_{\epsilon \rightarrow 0}$$

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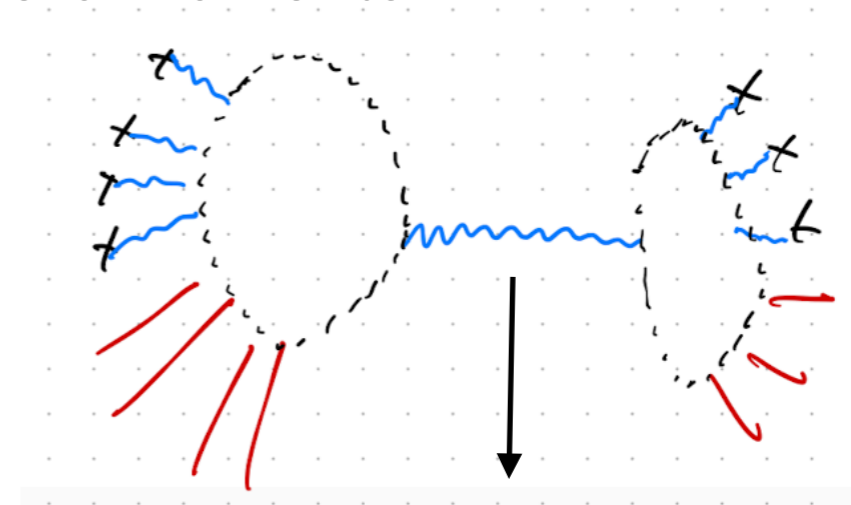
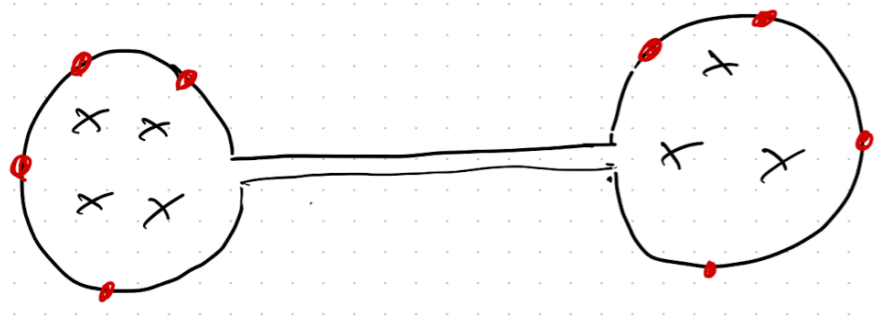
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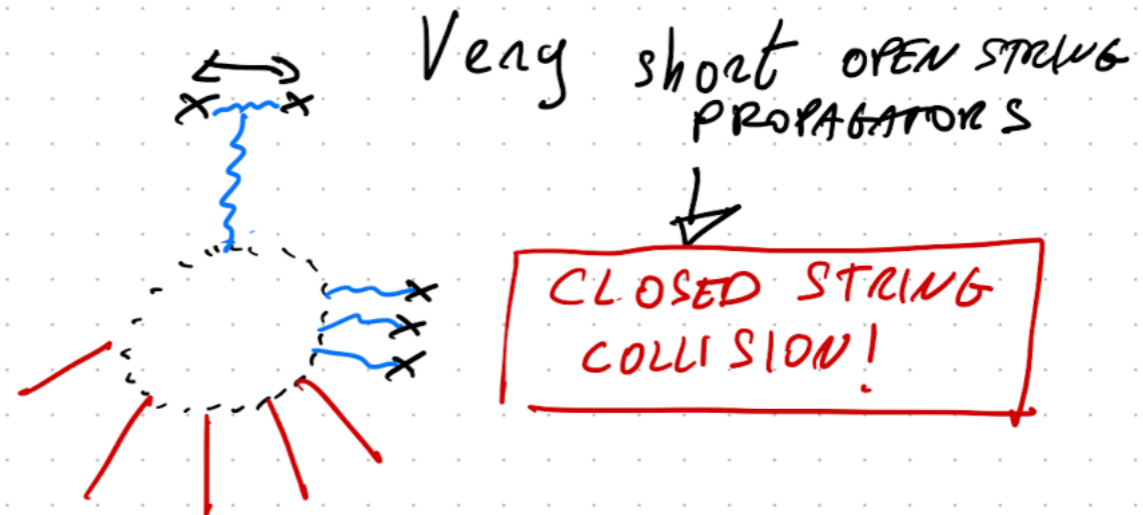
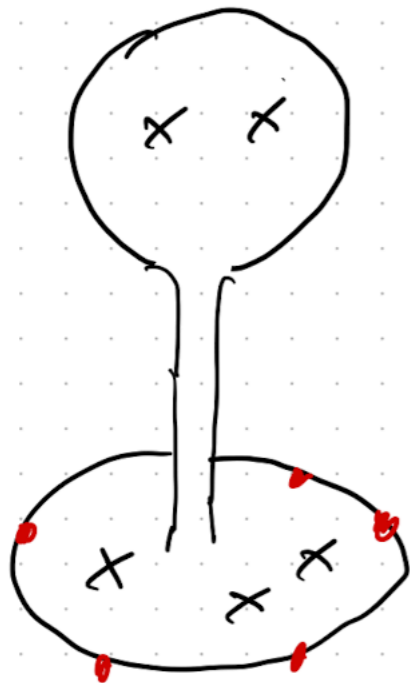


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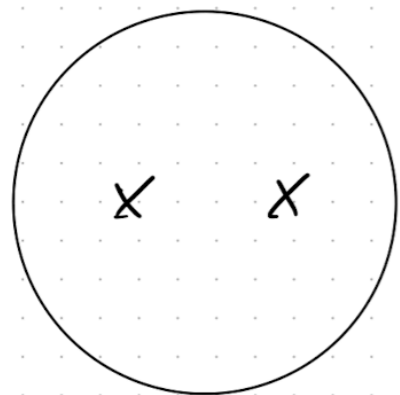
- This is enough to give well-defined amplitudes for generic external closed string momenta.
- This can be useful, for example, to study D-instanton contributions to closed string amplitudes, instead of the full-fledged open-closed SFT.

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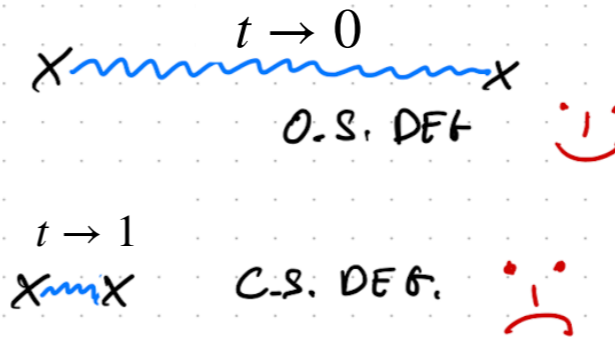
- However when closed strings are very soft (or zero momentum) the regions of closed string degenerations can produce un-tamed IR divergences



- Simplest example is the disk bulk two-point function



→ two degenerations



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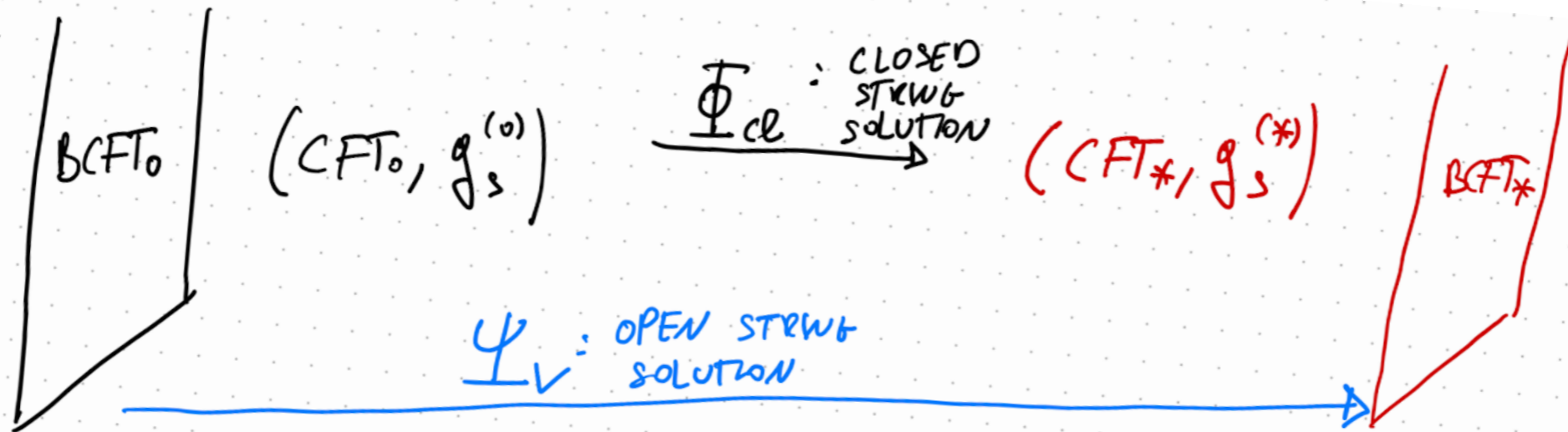
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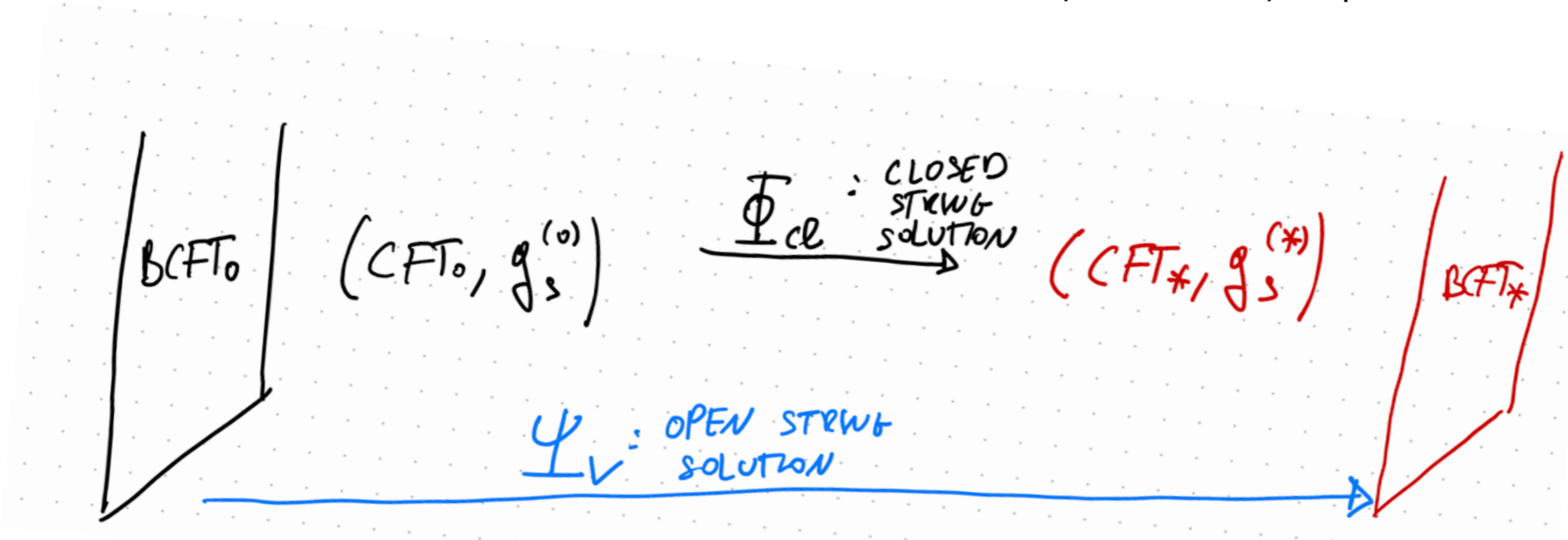
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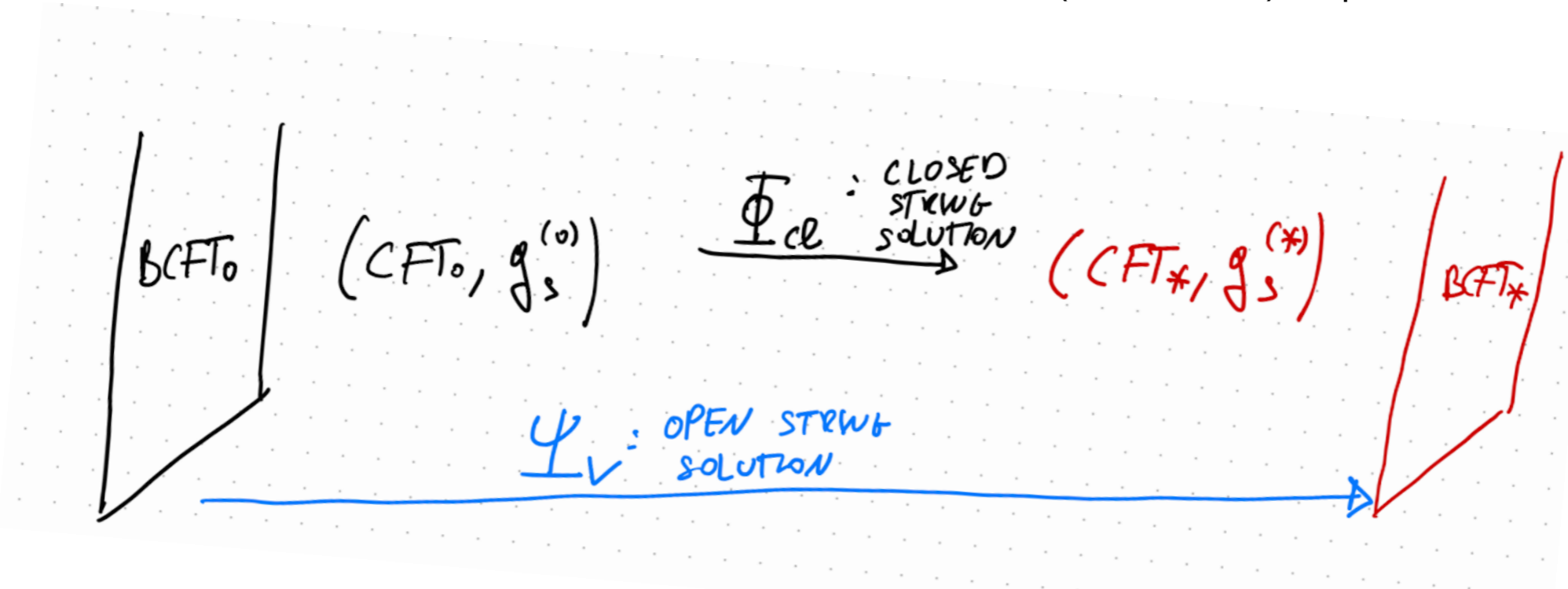


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- In this framework the D-branes are in probe approximation and don't back-react on the closed string background. (*interesting to relax this in the future!*)
- (Almost) equivalent question in CFT: how a boundary state changes if I change the bulk CFT? (*Fredenhagen-Gaberdiel 0609034, 2D "bulk-boundary" RG-flows*)

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- Example: D-brane deformations on a Narain compactification
- Conclusion and outlook

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- Level-matched closed string vector space, graded (degree=ghost-2), endowed with a symplectic form

$$\mathcal{H}^c : \quad b_0^- |\Phi\rangle = L_0^- |\Phi\rangle = 0,$$

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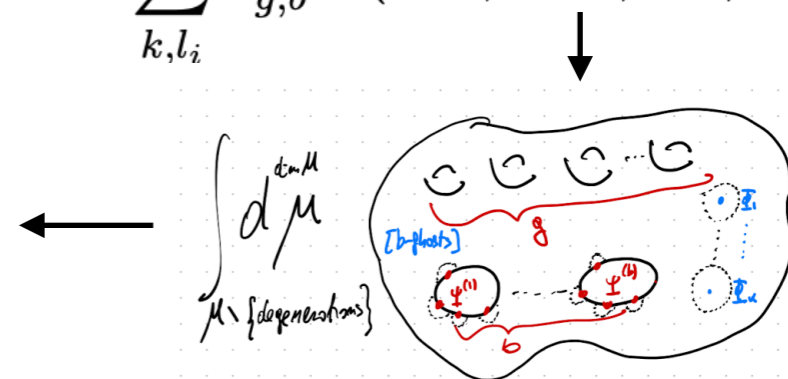
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Off-shell amplitude with integration near degenerations removed



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- Notice that  $S_{0,0}(\Phi)$  and  $S_{0,1}(0, \Psi)$  satisfy classical master equations. They are classical closed SFT and classical open SFT respectively.



- We can isolate the **genus zero** sector which obeys the master equation

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$$\boxed{(S_{\Phi_{cl}}, S_{\Phi_{cl}})_o = 0.}$$

- Zwiebach:** This should describe open strings in a deformed closed string background (always ignoring the D-brane back-reaction)

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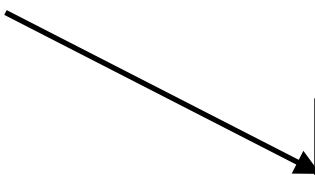
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**Notice in particular**

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**No OS inputs, CS output  
Not considered in usual Kajiura-Stasheff  
OCHA**

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- In particular there is a zero-product, a classical open string TADPOLE

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$$\delta\Lambda = \omega_{\text{c}}(\delta\Phi_{\text{cl}}, l_{00}^*) = -\omega_{\text{c}}(\Lambda, l_1^* l_{00}^*) = 0$$

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$$\boxed{l_1^* l_{00}^* = 0}$$

# Total cosmological constant

- The overall constant part of the action which is induced by the background shift is

$$\begin{aligned}\Lambda(\Phi_{\text{cl}}; \Psi_{\text{v}}) &= \Lambda_{\text{c}}(\Phi_{\text{cl}}) + \Lambda_{\text{o}}(\Phi_{\text{cl}}; \Psi_{\text{v}}) \\ &= -\frac{1}{g_s^{(0)}} \sum_{k=0}^{\infty} \frac{1}{(k+1)!} \omega_{\text{c}} \left( \Phi_{\text{cl}}, l_{k,0}(\Phi_{\text{cl}}^{\wedge k}) \right) + \\ &\quad -\frac{1}{g_s^{(0)}} \sum_{l=0}^{\infty} \sum_{k=0}^{\infty} \frac{1}{l+1} \frac{1}{k!} \omega_{\text{o}} \left( \Psi_{\text{v}}, m_{k,l}(\Phi_{\text{cl}}^{\wedge k}; \Psi_{\text{v}}^{\otimes l}) \right)\end{aligned}$$

- This quantity is (**trivially**) gauge invariant under **open string** gauge transformations (OS on-shell action)
- This quantity is (**non trivially**) gauge invariant under **closed string** gauge transformations.

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**Deformed BRST invariance  
of the deformed boundary state!**  
*(See Jakub talk!)*

# Background independence conjecture

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- The pair of solutions  $(\Phi_{\text{cl}}, \Psi_{\text{v}})$  should represent a new open-closed background  $(\text{CFT}_*, g_s^{(*)}; \|B_*\rangle\rangle)$

$$(\text{CFT}_0, g_s^{(0)}; \|B_0\rangle\rangle) \xrightarrow{(\Phi_{\text{cl}}, \Psi_{\text{v}})} (\text{CFT}_*, g_s^{(*)}; \|B_*\rangle\rangle)$$

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- For  $\Phi_{\text{cl}} = 0$  we get the good old “Sen’s conjecture” of OSFT

$$\Lambda(\Phi_{\text{cl}} = 0, \Psi_{\text{v}}) = \frac{1}{2\pi^2 g_s^{(0)}} \left( \langle 0 \| B_0 \rangle\rangle - \langle 0 \| B_* \rangle\rangle \right)$$



# **Perturbative background shifts: closed string solution**

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**Sufficient conditions  
for exact bulk marginality**

# Cosmological constant, closed string part

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- Closed string part of the cosmological constant

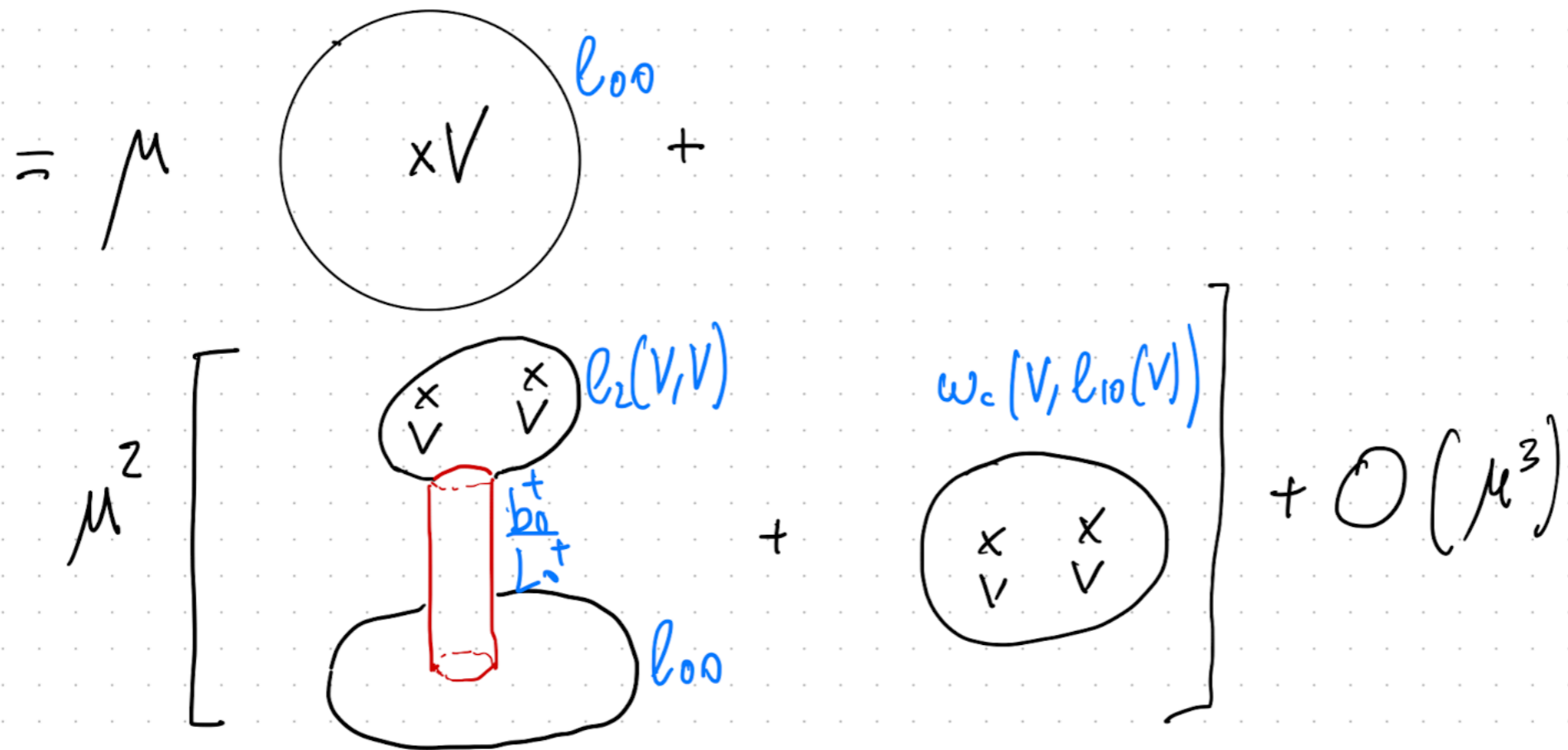
$$-g_s^{(0)} \Lambda_c(\mu) = \mu \omega_c(V, l_{0,0}) + \frac{1}{2} \mu^2 \left( \omega_c \left( l_{0,0}, \frac{b_0^+}{L_0^+} \bar{P}_0^+ l_2(V, V) \right) + \omega_c(V, l_{1,0}(V)) \right) + \mathcal{O}(\mu^3)$$



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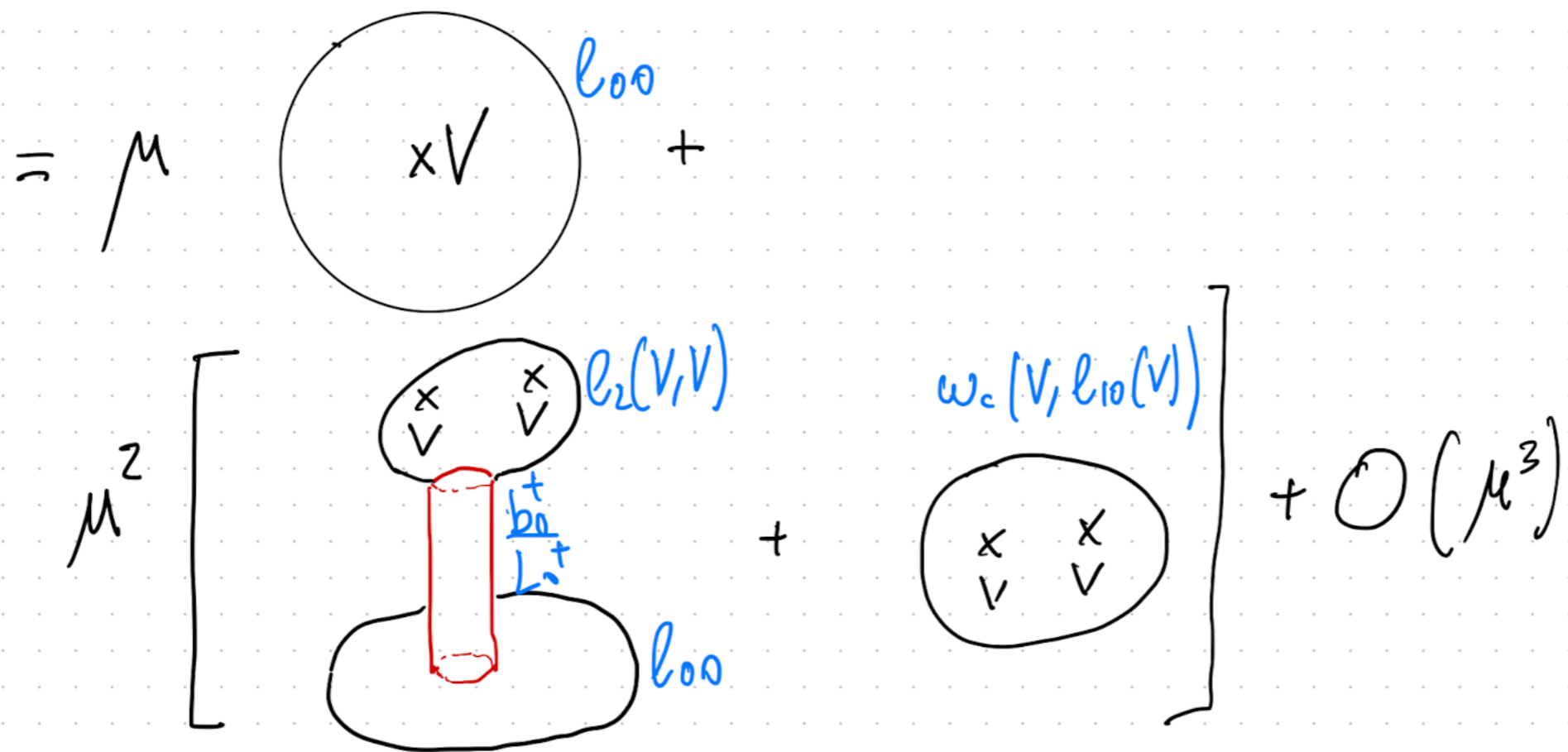
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- Remember: this alone is not gauge invariant! As an amplitude it is missing the region of open string degeneration.

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$$\tilde{m}_0 + \tilde{m}_1(\Psi) + \tilde{m}_2(\Psi, \Psi) + \dots = 0.$$

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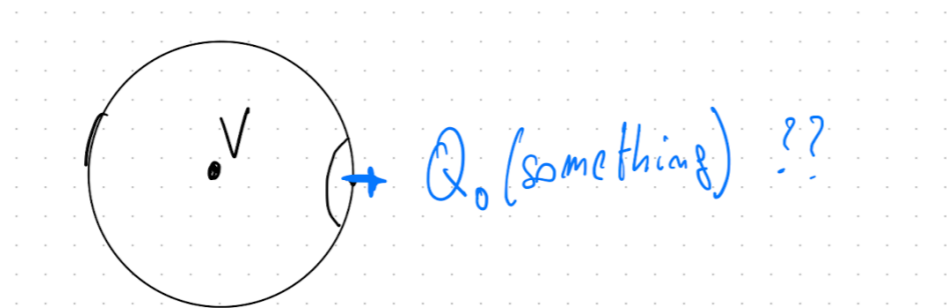
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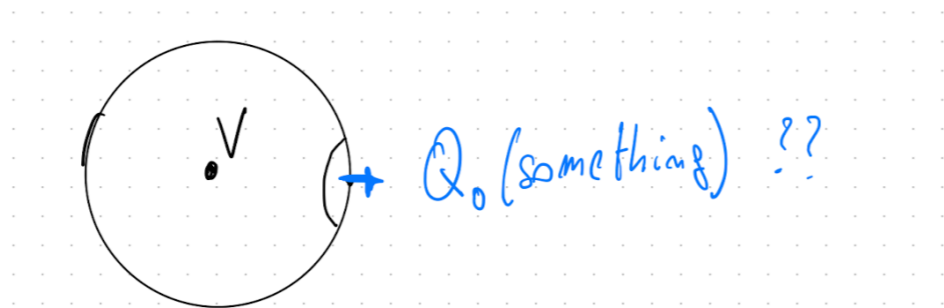
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**Strong condition  
on bulk-boundary OPE!**

**See 2103.04919 for examples**



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⋮

- EFT interpretation: *the “massless” tadpole is zero . AKA: amplitudes between deforming closed strings and a single massless open string are zero*

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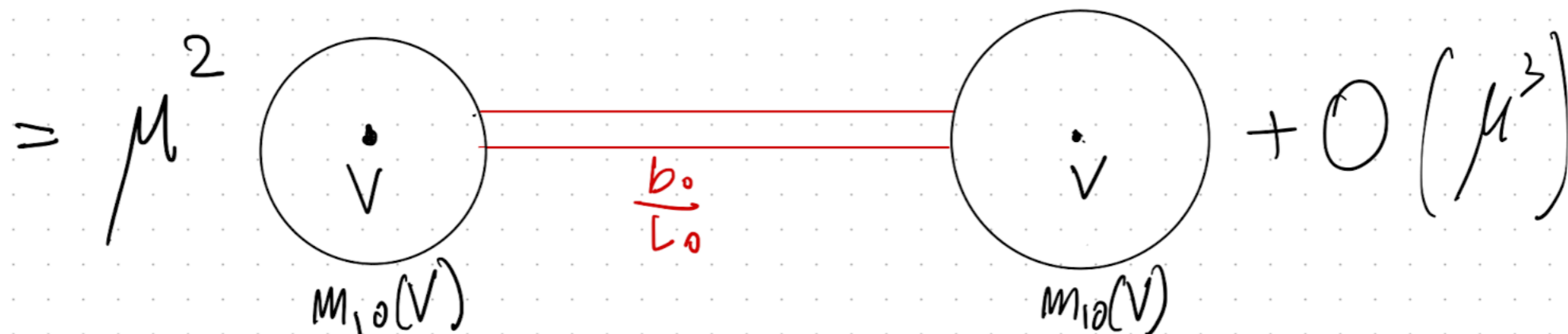
- The open string contribution to the cosmological constant is

$$-g_s^{(0)} \Lambda_o(\mu) = \frac{1}{2} \mu^2 \omega_o \left( m_{1,0}(V), \frac{b_0}{L_0} \bar{P}_0 m_{1,0}(V) \right) + \mathcal{O}(\mu^3)$$

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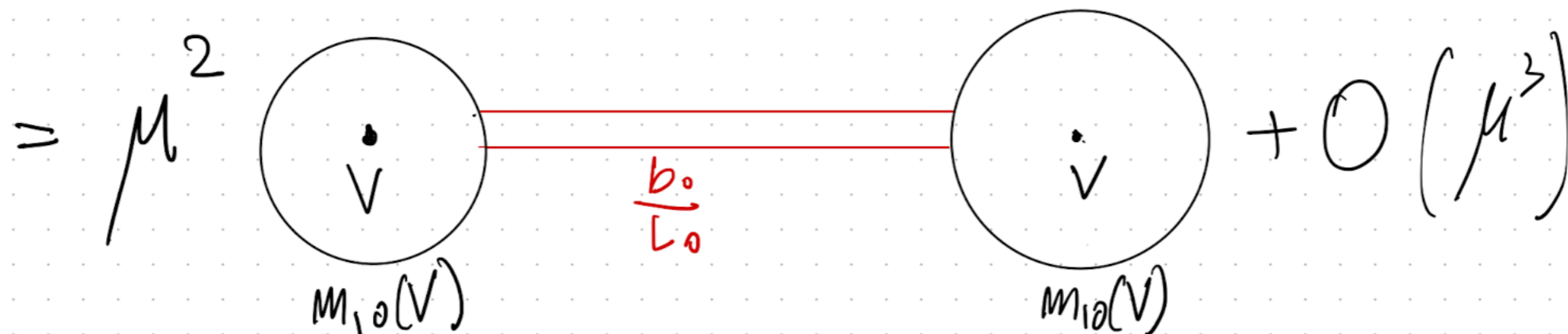
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- This is giving the missing part containing open string degeneration!



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- Adding the closed and the open contribution we find

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- At order  $\mu^n$  we find an n-point bulk disk amplitude with correct treatment of IR divergences
- Notice: the two-point amplitude

$$\mathcal{A}(V_1, V_2) = \omega_c\left(l_{0,0}, \frac{b_0^+}{L_0^+}\bar{P}_0^+ l_2(V_1, V_2)\right) + \omega_c(V_1, l_{1,0}(V_2)) + \\ + \omega_o\left(m_{1,0}(V_1), \frac{b_0}{L_0}\bar{P}_0 m_{1,0}(V_2)\right)$$

correctly decouples BRST exact states ( $V_2 = Q(\Lambda)$ ) (up to boundary terms) thanks to the non-trivial SDHA relation

$$[Q_c, l_{1,0}] + l_2 l_{0,0} + l_{0,1} m_{1,0} = 0$$

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$$-\omega_o \left( m_{1,0}(V), \frac{b_0}{L_0} \bar{P}_0 m_{1,0}(V) \right) - \omega_c \left( l_{0,0}, \frac{b_0^+}{L_0^+} \bar{P}_0^+ l_2(V, V) \right) = \frac{1}{\pi^2} \left( \int_0^{1/\lambda_o^2} ds s^{\epsilon_o} + \int_{1/\lambda_o^2}^1 ds y(s)^{\epsilon_c} \right) (1 - s^2) \times$$

$$\times \langle \mathbb{V}_{1,1}(i, \bar{i}) \mathbb{V}_{1,1}(is, \bar{is}) \rangle_{\text{UHP}} \Bigg|_{\substack{\epsilon_c \rightarrow 0 \\ \epsilon_o \rightarrow 0}}$$



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 -\omega_o \left( m_{1,0}(V), \frac{b_0}{L_0} \bar{P}_0 m_{1,0}(V) \right) - \omega_c \left( l_{0,0}, \frac{b_0^+}{L_0^+} \bar{P}_0^+ l_2(V, V) \right) &= \frac{1}{\pi^2} \left( \int_0^{1/\lambda_o^2} ds s^{\epsilon_o} + \int_{1/\lambda_o^2}^1 ds y(s)^{\epsilon_c} \right) (1 - s^2) \times \\
 &\times \langle \mathbb{V}_{1,1}(i, \bar{i}) \mathbb{V}_{1,1}(is, \bar{is}) \rangle_{\text{UHP}} \Bigg|_{\substack{\epsilon_c \rightarrow 0 \\ \epsilon_o \rightarrow 0}}
 \end{aligned}$$

$y = \frac{1 - \sqrt{s}}{1 + \sqrt{s}}$   
 $\uparrow$

## Explicit evaluation up to 2nd order

- We constructed  $SL(2, \mathbb{C})$  open-closed vertices for which  $l_{1,0} = 0$

$$[Q_c, l_{1,0}] + l_2 l_{0,0} + l_{0,1} m_{1,0} = 0 \quad l_{1,0} = 0 \quad \rightarrow \quad l_2 l_{0,0} + l_{0,1} m_{1,0} = 0.$$

- We considered a “standard” closed string of the form

$$V(z, \bar{z}) = c\bar{c}\mathbb{V}_{1,1}(z, \bar{z})$$

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- Witten Theory:  $\lambda_o \rightarrow 1^-$ , the closed string channel disappears, together with the regularization at closed string degeneration. **Singular limit!**

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- To compare with the change in the g-function we have to relate the SFT parameter  $\epsilon$  with the sigma-model one  $\epsilon_\sigma$

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**CONSISTENCY CHECK**

***This is independent of boundary conditions!***  $\tilde{\epsilon}_{ij} = \epsilon_{ik} (\Omega^T)^k_j, \quad \Omega g \Omega^T = g$

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- ***Thanks, comments and suggestions are welcome!!***