RR-fields for the RNS-worldline

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I) Motivation

II) Spin fields on the world line

III) BRST cohomology

IV) Theory of background fields

V) sigma model description

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1) Motivation:

- SFT as a theory of background fields Φ

$$\left(Q_{BRST} [\Phi] \right)^2 \equiv 0$$

$$\Rightarrow \mathcal{F}(D.\Phi) = 0$$

- Nilpotence of the pure spinor BRST op.

$\Rightarrow$ e.m. for massless NSNS fields

(N. Berkovits, P. Howe '02)

- off-shell RNS construction of Q_B is difficult
  $\rightsquigarrow$ extract e.m. from β-fn or S-matrix.

- RNS-point particle: $Q_B^2 = 0 \Rightarrow$
  Einstein / Dilaton / Kalb-Ramond e.m.
  (R. Bonezzi, A. Meyer, I.S. '18)

<u>Question:</u> RR-e.m. from world-line ?

2) Prologue: R-sector in non-critical dim's

$$V_{\underline{\alpha}} \sim S_A = e^{\frac{1}{2}\underline{\alpha}\cdot\underline{H}} \quad ; \quad \underline{\alpha} = (\pm, \pm, \pm, \cancel{\pm})$$

$$\langle V_{\underline{\alpha}}(z)\, V_{\underline{\alpha}'}(0) \rangle \sim \frac{1}{\sqrt{z}}$$

$\rightsquigarrow$ VOA does not give rise to a Lie (Poisson) algebra.

We will encounter a similar feature for the R-sector on the world line.

3) World line $\quad \mathcal{N}=2 \quad (\psi^\mu, \bar\psi^\mu \sim \psi_{1/2}, \psi_{-1/2})$

- $$Q_B = c\,\underline{P}^2 + \gamma \underset{\frac{\partial}{\partial\psi^\mu}P_\mu}{\bar q} + \overset{\frac{\partial}{\partial\beta}}{\bar\gamma}\, \overset{\sim\,\psi^\mu P_\mu}{q} + b\gamma\bar\gamma$$

- $$\mathcal{H} = \mathrm{Span}\{\underbrace{\Psi(x^\mu, c, \gamma, \beta, \psi^\mu)}_{=\,\delta Q\beta\ \rightsquigarrow\ \boxed{\text{Operator-state correspondence}}}\}$$

- $$P_\mu \rightsquigarrow \Pi_\mu = P_\mu + A_\mu : \{Q_B, Q_B\} = 0 \Rightarrow \text{Yang Mills}$$

(Dai, Huang, Siegel '08)

$\underline{\mathcal{N}=1}$: $(\psi^\mu \sim \psi_0^\mu \sim \Gamma^\mu)$

- $\Phi = \Phi_{\underline{\alpha}} |\underline{\alpha}\rangle = \delta Q |?\rangle$

3) "Bosonization": $(D=4,\ \underline{\alpha} = (\alpha\ \dot{\alpha}))$

$$\psi^\mu := \left(\vartheta^\alpha \sigma^\mu_{\alpha\dot{\alpha}} \tilde{\lambda}^{\dot{\alpha}} + \tilde{\vartheta}_{\dot{\beta}} \tilde{\sigma}^{\mu\,\dot{\beta}\beta} \lambda_\beta\right), \quad \lambda_\alpha = \uparrow \frac{\partial}{\partial \theta^\alpha}$$

$\uparrow$: shifts grassmann degree (mod 2)

- $\{\theta^\alpha, \lambda_\alpha, \uparrow\}$ does not form a Lie algebra!

Relations: $\{\theta^\alpha, \theta^\beta\} = 0$

$[\lambda_\alpha, \theta^\beta] = \delta_\alpha{}^\beta \uparrow$

Then, with $|\alpha\rangle = \theta^\alpha$ (or $\theta^\alpha\uparrow$),

$$\{\psi^\mu, \psi^\nu\}\,\Phi = 2\eta^{\mu\nu}\,\Phi \quad \checkmark$$

$$\rightsquigarrow \quad Q_0 = -cH_0 + \gamma q_0 + b\gamma^2 , \qquad q_0 = \psi^\mu p_\mu$$

reproduces the standard cohomology:

$$|\Phi\rangle = \varphi^\uparrow_\alpha(x)\vartheta^\alpha\uparrow + \tilde{\chi}^{\uparrow\dot{\alpha}}(x)\tilde{\vartheta}_{\dot{\alpha}}\uparrow , \qquad \not{\partial}\tilde{\chi}^\uparrow = \tilde{\not{\partial}}\varphi^\uparrow = 0.$$

4) Chiral Theory:

Rep: $$q_0 = \mathbf{q} + \bar{\mathbf{q}} = \Psi^\mu p_\mu + \bar{\Psi}^\mu p_\mu = \tilde{\vartheta}_{\dot{\beta}} \tilde{\sigma}^{\mu\ \dot{\beta}\beta} \lambda_\beta \uparrow p_\mu + \vartheta^\alpha \sigma^\mu_{\alpha\dot{\alpha}} \tilde{\lambda}^{\dot{\alpha}} \uparrow p_\mu$$

- $\bar{\mathbf{q}}$ and $\mathbf{q}$ are separately nilpotent

  coh($\mathbf{q}$) = coh (Q_0) (c.f pure spinor)

Deformations:

$$\delta \mathbf{q} = s\, \tilde{\vartheta}_{\dot{\beta}} \tilde{A}^{\dot{\beta}\beta} \lambda_\beta$$: coupling to Y-M backgnd

$$\delta \mathbf{q} = s\, \vartheta^\beta F_{\beta\dot{\beta}} \tilde{\lambda}^{\dot{\beta}}$$: sets anti-chiral fermions to zero

$$\delta \mathbf{q} = s\, \tilde{\vartheta}^{\dot{\beta}} \tilde{F}_{\dot{\beta}\dot{\gamma}} \tilde{\lambda}^{\dot{\gamma}}$$: mixes both chiralities

5) <u>RR - fields</u>: are contained in a ψ^μ-invariant subspace of 2-particle states:

$$|\dot\alpha\dot\beta\rangle = (\tilde\vartheta_{\dot\alpha}\tilde\vartheta_{\dot\beta} - \tilde\vartheta_{\dot\alpha}\uparrow\tilde\vartheta_{\dot\beta}\uparrow)\begin{pmatrix}|0\rangle\\ \uparrow|0\rangle\end{pmatrix} \quad \text{and} \quad |e^{\beta}_{\ \dot\gamma}\rangle = (\vartheta^\beta\uparrow\tilde\vartheta_{\dot\gamma}\uparrow - \vartheta^\beta\tilde\vartheta_{\dot\gamma})\begin{pmatrix}|0\rangle\\ \uparrow|0\rangle\end{pmatrix}$$

$$\ker(q) = \left\{ |F,\tilde F\rangle = \underbrace{F^{[1]\dot\beta}_{\alpha}\,|e^{\alpha}_{\ \dot\beta}\rangle + F^{[2]}{}_{\alpha\beta}\,|\alpha\beta\rangle}_{=|F\rangle} + \underbrace{\tilde A^{[1]\dot\alpha}{}_{\beta}\,|e^{\beta}_{\ \dot\alpha}\rangle + \tilde F^{[2]\dot\alpha\dot\beta}\,|\dot\alpha\dot\beta\rangle}_{=q|G\rangle} \right\}$$

$$\boxed{dF = \delta F = 0 \;:\quad \text{(linear) e. m. for RR-fields}}$$

6) RR-Background fields:

- Operator-state correspondence:

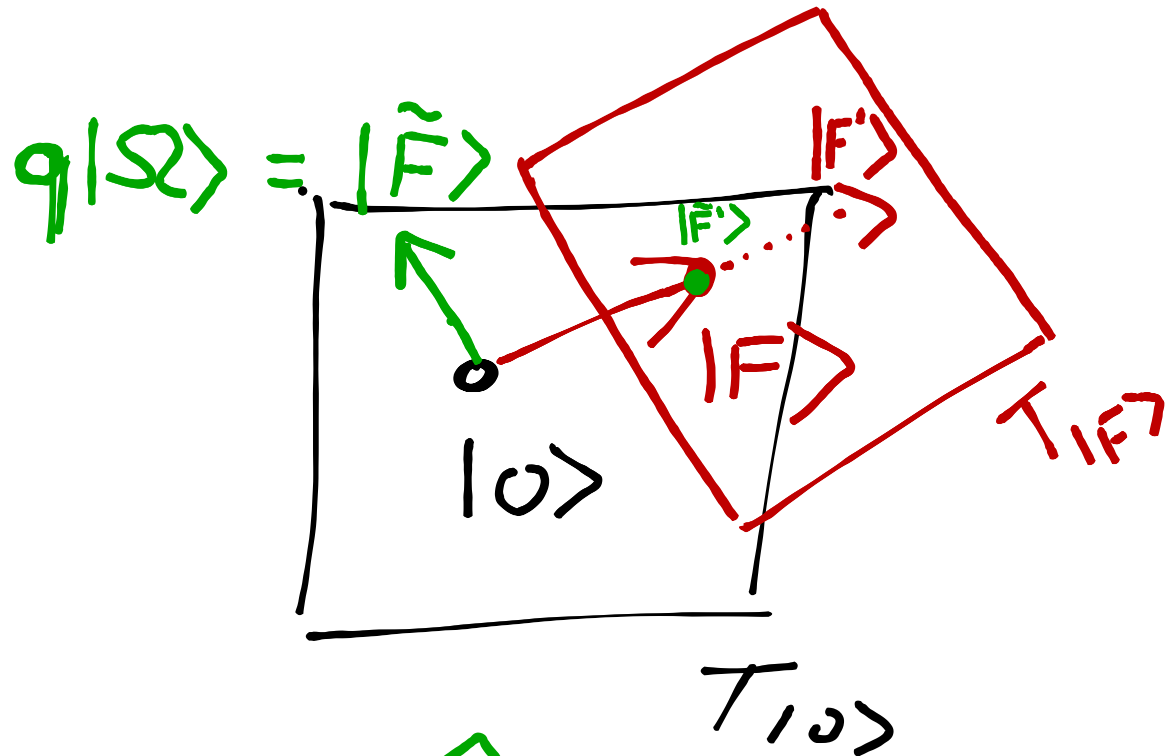
$$\delta\mathbf{q} = F_{\alpha\dot{\beta}}\vartheta^{\alpha}\tilde{\lambda}^{\dot{\beta}} + F_{\alpha\beta}\vartheta^{\alpha}\lambda^{\beta}$$

$$|F\rangle = F_{\alpha}{}^{\dot{\beta}}\,|e^{\alpha}{}_{\dot{\beta}}\rangle + F_{\alpha\beta}\,|\alpha\beta\rangle = \boldsymbol{\delta}\mathbf{q}\,|\Omega\rangle$$

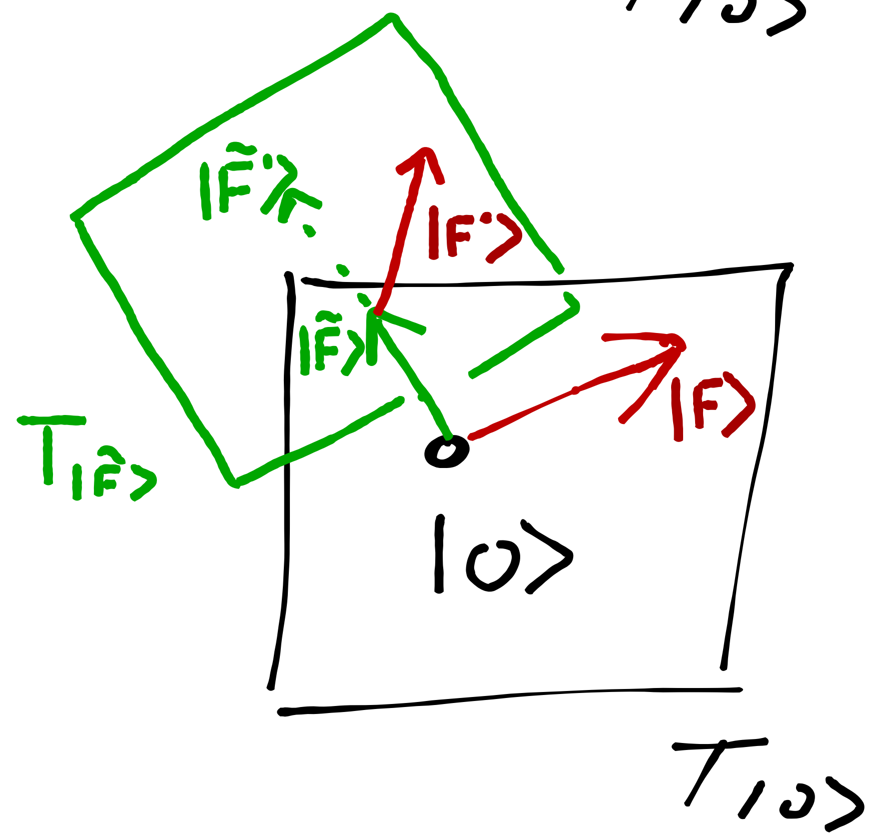
$$= \left(\tilde{\vartheta}_{\dot{\gamma}}\uparrow\tilde{\vartheta}^{\dot{\gamma}} - \tilde{\vartheta}_{\dot{\gamma}}\tilde{\vartheta}^{\dot{\gamma}}\uparrow + \vartheta^{\gamma}\uparrow\vartheta_{\gamma} - \tilde{\vartheta}_{\dot{\gamma}}\vartheta_{\gamma}\uparrow\right)$$

$\rightsquigarrow$ identify background field for a given state.

$$\delta\mathbf{q} = s\, F_{\alpha\dot\beta}\vartheta^{\alpha}\tilde\lambda^{\dot\beta}\,, \quad s \ll 1$$



$$\delta\mathbf{q} = s\, \tilde\vartheta_{\dot\beta}\tilde F^{\dot\beta\beta}\lambda_{\beta}\,, \quad s \ll 1$$



- non-linear e.m. do not derive from an action.

7) World line model

- $\ker(q = \tilde{\vartheta}\not{p}\lambda) = \ker(\tilde{C}^{\dot\alpha} = \tilde{p}^{\dot\alpha\alpha}\lambda_\alpha)$
- change variables: $\vartheta^\alpha \uparrow \longrightarrow \vartheta^\alpha$

$$S = \int \left(p_\mu \dot{x}^\mu + \vartheta^\alpha \dot{\lambda}_\alpha + \tilde{\vartheta}_{\dot\alpha} \dot{\tilde{\lambda}}^{\dot\alpha} + \tilde{w}_{\dot\alpha} \tilde{p}^{\dot\alpha\alpha} \lambda_\alpha + w^\alpha p_{\alpha\dot\alpha} \tilde{\lambda}^{\dot\alpha} \right) \mathrm{d}\tau$$

Lagrange mult.

- BRST quantization:

$$S = \int \left(p_\mu \dot{x}^\mu + \vartheta^\alpha \dot{\lambda}_\alpha + \tilde{\vartheta}_{\dot\alpha} \dot{\tilde{\lambda}}^{\dot\alpha} + \omega_\beta \dot{\tau}^\beta + \tilde{\omega}^{\dot\beta} \dot{\tilde{\tau}}_{\dot\beta} \right) \mathrm{d}\tau$$

(anti) ghost

BRST operator: $\mathcal{O} = \underbrace{\tau^\alpha C_\alpha}_{= \mathcal{O}^{(L)}} + \tilde{\tau}_{\dot\alpha} \tilde{C}^{\dot\alpha}$

wave functions: $\psi(x, \vartheta^\alpha, \tilde{\tau}_{\dot\alpha})$

RR 1-Form: $|F\rangle = F_\alpha{}^{\dot\beta} \vartheta^\alpha \tilde{\tau}_{\dot\beta} \in \mathrm{coh}(\mathcal{O}) \Rightarrow dF = \delta F = 0$

RR - e.m. ✓

- no operator-state correspondence: $\lightning$ theory of background fields

Relation to the RNS-formulation:

Rep: $$S = \int \left(p_\mu \dot{x}^\mu + \vartheta^\alpha \dot{\lambda}_\alpha + \tilde{\vartheta}_{\dot{\alpha}} \dot{\tilde{\lambda}}^{\dot{\alpha}} + \omega_\beta \dot{\tau}^\beta + \tilde{\omega}^{\dot{\beta}} \dot{\tilde{\tau}}_{\dot{\beta}} \right) \mathrm{d}\tau$$

+ constraint: $$\Delta S = \int \tilde{\rho}_{\dot{\alpha}} (\tilde{\omega}^{\dot{\alpha}} - \tilde{\lambda}^{\dot{\alpha}} \uparrow) + \rho^\alpha (\omega_\alpha - \lambda_\alpha \uparrow) \equiv \int \tilde{\rho}_{\dot{\alpha}} \tilde{\Theta}^{\dot{\alpha}} + \rho^\alpha \Theta_\alpha$$

$\rightsquigarrow$ secondary constraint: $\{\mathcal{O}, \tilde{\Theta}^{\dot{\alpha}}\} = \tilde{C}^{\dot{\alpha}}$

- inhomogeneous ghost number!

- solve $\tilde{\Theta}^{\dot\alpha} = 0 \Rightarrow \tilde{\vartheta}_{\dot\alpha} - \tilde{\tau}_{\dot\alpha} \uparrow$ is cyclic

$\Rightarrow \quad |F\rangle = F_\alpha{}^{\dot\beta}\vartheta^\alpha\tilde{\tau}_{\dot\beta} = F_\alpha{}^{\dot\beta}|e^\alpha{}_{\dot\beta}\rangle$ ✓

Relation to the Brink-Schwarz particle:

- $\lambda_\alpha, \tilde{\lambda}^{\dot\alpha}$ and $\vartheta_\alpha, \tilde{\vartheta}^{\dot\beta}$ are interpreted as ghosts and anti-ghost while $(\omega_\alpha, \tau^\alpha, \tilde{\tau}_{\dot\alpha}, \tilde{\omega}^{\dot\beta})$ are world-line fermions.

+ change of variables:

$$\omega_\alpha = p_{\alpha\dot\alpha}\tilde{\theta}^{\dot\alpha} \quad \text{and} \quad \tilde{\omega}^{\dot\alpha} = \tilde{p}^{\dot\alpha\alpha}\theta_\alpha$$

Then

$$S = \int p_{\alpha\dot{\alpha}}\left(-\frac{1}{2}\dot{x}^{\dot{\alpha}\alpha} + \tilde{\theta}^{\dot{\alpha}}\dot{\tau}^{\alpha} + \dot{\tilde{\tau}}^{\dot{\alpha}}\theta^{\alpha}\right)\mathrm{d}t$$

with invariance:

$$\begin{aligned}
\delta x^{\dot{\alpha}\alpha} &= 2\tilde{\tau}^{\dot{\alpha}}\epsilon^{\alpha} + 2\tilde{\epsilon}^{\dot{\alpha}}\tau^{\alpha} \\
\delta\tilde{\theta}^{\dot{\alpha}} &= \tilde{\epsilon}^{\dot{\alpha}} \\
\delta\theta_{\alpha} &= \epsilon_{\alpha}\,,
\end{aligned}$$

- ~ κ-symmetry
- no space-time SUSY!

Summary:

- RR - background in (4 dim) "world line" ✓
- sigma model not fully constructed
- 10- dimension / relation to pure spinor