Higher Spin Gravities: News and (possible) relations to string theory

String Field Theory, Prague

Evgeny Skvortsov, UMONS

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FREEDOM TO RESEARCH

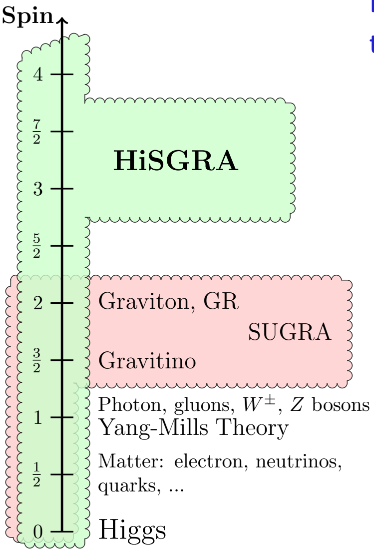
Main Messages

- Higher Spin Gravities (HiSGRA) — the most minimal extensions of gravity with massless higher spin fields — toy models of Quantum Gravity. The idea is that massless fields $\rightarrow$ gauge fields; more gauge symmetries $\rightarrow$ less counterterms $\rightarrow$ Quantum Gravity. No free lunch: HiSGRA are hard to construct and there are very few (no-go's)
- Some HiSGRA: $3d$, conformal, chiral, ...
- HiSGRA have a niche within AdS/CFT as duals of (Chern-Simons) vector models, relation to $3d$ bosonization duality
- A number of conjectural relations between HiSGRA and string theory
- Techniques: strong homotopy algebras, higher spin symmetries
- **More:** Snowmass paper, ArXiv: 2205.01567; **even more:** lectures by Dmitry Ponomarev ArXiv: 2206.15385

Plan/keywords

- Spin, higher spin, higher spin gravity
- Power of higher spin symmetry;
  how not to construct HiSGRA: no-go's, ...
- HiSGRA via AdS/CFT lenses;
  possible relations to string theory
- All HiSGRA's in 2022
- Chiral HiSGRA and $3d$ bosonization duality
- Higher spin symmetry as 'new Virasoro'
- Star-products, Strong homotopy algebras, etc.

Spin by spin



Different spins lead to very different types of theories/physics:

- $s = 0$: Higgs
- $s = 1/2$: Matter

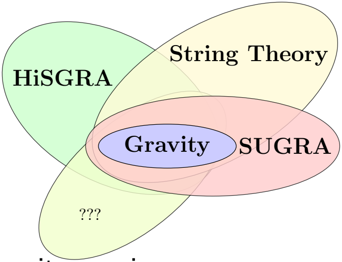
- $s = 1$: Yang-Mills, Lie algebras
- $s = 3/2$: SUGRA and supergeometry, graviton $\in$ spectrum
- $s = 2$ (graviton): GR and Riemann Geometry, no color

- $s > 2$: HiSGRA and String theory, ∞ states, graviton is there too!

Why higher spins?

Various examples (not all)

- string theory
- divergences in (SU)GRA's
- Quantum Gravity via AdS/CFT



seem to indicate that quantization of gravity requires

- infinitely many fields
- for any $s > 0$ a spin-s field must be part of the spectrum

HiSGRA is to find the most minimalistic extension of gravity by massless, i.e. gauge, higher spin fields. Vast gauge symmetry should render it finite.

Quantizing Gravity via HiSGRA = Constructing Classical HiSGRA?

What Higher Spin Problem is: Field theory approach

A massless spin-s particle can be described by a rank-s tensor

$$\delta\Phi_{\mu_1\ldots\mu_s} = \nabla_{\mu_1}\xi_{\mu_2\ldots\mu_s} + \text{permutations}$$

which generalizes $\delta A_\mu = \partial_\mu \xi, \qquad \delta g_{\mu\nu} = \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu$
Fronsdal, Berends, Burgers, Van Dam, Bengtsson^2, Brink, ...

Problem: find a nonlinear completion (action, gauge symmetries)

$$S = \int (\nabla\Phi)^2 + \mathcal{O}(\Phi^3) + \ldots \qquad\qquad \delta\Phi_{\ldots} = \nabla_{.}\xi_{\ldots} + \ldots$$

and prove that it is UV-finite, hence a Quantum Gravity model

Warning: brute force does not seem to work!

Power of higher spin symmetry know your friend/enemy

too small symmetry: nothing can be computed even with a theory

too big symmetry: everything is fixed even without a theory

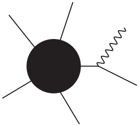
higher spin symmetry: almost everything is fixed and there are very few theories

Higher spin symmetry constraints on S-matrix

Let's study **asymptotic higher spin symmetry** in Minkowski

$$\delta\Phi_{\mu_1\dots\mu_s}(x) = \partial_{\mu_1}\xi_{\mu_2\dots\mu_s}$$

Weinberg low energy theorem (similarly, Coleman-Mandula theorem):



$$\sum_i g_i\, p^i_{\mu_1}\dots p^i_{\mu_{s-1}} = 0$$

- $s=1$, charge conservation $\sum g_i$=0
- $s=2$, equivalence principle $\sum g_i\, p^i_\mu$=0 $\rightarrow$ $g_i = g$
- $s>2$, too many conservation laws and **HiSGRA** $\boldsymbol{S=1}$ 

May be massless higher spin fields confine? or do not exist?

Local vs. Global tension

Global picture: The S-matrix has to be trivial, $S = 1$, whenever there is at least one massless higher spin $s > 2$ particle

Local picture: The same time, for every triplet of helicities, $\lambda_{1,2,3}$ there is a nontrivial cubic vertex/amplitude (Brink, Bengtsson^2, Linden; ...)

$$\boldsymbol{V^{\lambda_1,\lambda_2,\lambda_3} \sim [12]^{\lambda_1+\lambda_2-\lambda_3}[23]^{\lambda_2+\lambda_3-\lambda_1}[13]^{\lambda_1+\lambda_3-\lambda_2}} \oplus \text{c.c.}$$

Puzzle: Why do we have cubic amplitudes for all possible helicities, but do not seem to have theories that apply those?

Possible resolutions: (1) that's life and there may not exist any HiSGRA, e.g. there are obstructions at the quartic order (Bekaert, Boulanger, Leclerq; Roiban, Tseytlin; Taronna; Ponomarev, E.S.; ...); (2) **there are some (very few!) HiSGRA's, but the interactions are fine-tuned to give $S = 1$ in Minkowski. There are other backgrounds, for example AdS, where $S \neq 1$**

Higher spin symmetry constraints on holographic S-matrix

Let's study **asymptotic higher spin symmetry** in anti-de Sitter

$$\delta\Phi_{\mu_1\dots\mu_s}(x) = \nabla_{\mu_1}\xi_{\mu_2\dots\mu_s} \qquad \Longleftrightarrow \qquad \partial^m J_{ma_2\dots a_s} = 0$$

$$\Updownarrow$$

free CFT

Given a CFT in $d \geq 3$ with stress-tensor J_2 and at least one higher-spin current J_s, one can prove that it is a free CFT in disguise Maldacena, Zhiboedov; Boulanger, Ponomarev, E.S., Taronna; Alba, Diab, Stanev

This essentially proves the duality no matter how the bulk theory is realized. It may not even exist!

This is a generalization of the Weinberg and Coleman-Mandula theorems to AdS/CFT: higher spin symmetry implies

holographic HiSGRA $S =$ **free CFT**

Higher spin symmetry constraints on holographic S-matrix

Let's study **asymptotic slightly-broken higher spin symmetry** in anti-de Sitter

$$\delta\Phi_{\mu_1\ldots\mu_s}(x) = \nabla_{\mu_1}\xi_{\mu_2\ldots\mu_s} \qquad \Longleftrightarrow \qquad \partial^m J_{ma_2\ldots a_s} = \frac{1}{N}[JJ] \neq \mathbf{0}$$

Large-N critical vector model (Wilson-Fisher)

$$S = \int d^3x \left((\partial\phi^i)^2 + \frac{\lambda}{4!}(\phi^i\phi^i)^2 \right)$$

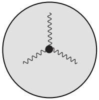
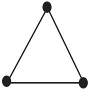
should be dual to the same HiSGRA for $\Delta = 2$ boundary conditions on $\Phi(x)$ (Klebanov, Polyakov). **This duality is kinematically related to the first one** (Hartman, Rastelli; Giombi, Yin; Bekaert, Joung, Mourad).

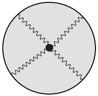
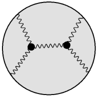
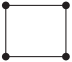
holographic HiSGRA $S =$ Large-N Ising

This can be extended to Chern-Simons Matter theories, (Chang, Minwalla, Sharma, Yin, Giombi, Prakash, Trivedi, Wadia; Aharony; Maldacena, Zhiboedov, ...)
New input: vector models have slightly-broken higher spin symmetry!

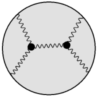
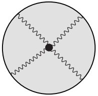
Where Field theory cries...

Ingenious: invert AdS/CFT and reconstruct the dual theory from free CFT (Bekaert, Erdmenger, Ponomarev, Sleight; Taronna, Sleight)

$\#$  $= \#$  Costa et al

 $+$  $+\, t, u =$  $= \langle JJJJ \rangle$

But there is a big 'but' lurking around the corner:

 $+\, t, u = \mathbf{2} \langle JJJJ \rangle$  $= -\langle JJJJ \rangle \sim \Phi^2 \dfrac{1}{\Box + \Lambda} \Phi^2$

Quartic vertex $\sim$ exchange. Field theory does not like that. Stringy?

There are similar obstructions in flat space (Bekaert, Boulanger, Leclerq; Roiban, Tseytlin; Taronna; Ponomarev, E.S.; ...) **No large gap, so as expected!**

S-matrix summary

We see that **asymptotic higher spin symmetries** (HSS)

$$\delta\Phi_{\mu_1\dots\mu_s}(x) = \nabla_{\mu_1}\xi_{\mu_2\dots\mu_s}$$

seem to completely fix (holographic) S-matrix to be

$$S_{\text{HiSGRA}} = \begin{cases} 1^{***}, & \text{flat space} \\ \text{free CFT}, & \text{asymptotic AdS, unbroken HSS} \\ \textbf{Chern-Simons Matter}, & \textbf{asymptotic AdS}_4\textbf{, slightly-broken HSS} \end{cases}$$

Trivial/known S-matrix can still be helpful for QG toy-models

The most interesting applications are for AdS_4/CFT_3 and three-dimensional dualities (power of HSS is underexplored)

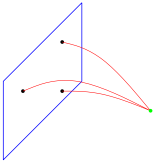
Both Minkowski and AdS cases reveal certain non-localities to be tamed. HSS mixes ∞ spins and derivatives, invalidating the local QFT approach

What Higher Spin Problem is: AdS/CFT

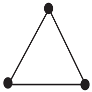
In the context of AdS/CFT: gauge fields are dual to conserved tensors

$$\partial^m J_{ma_2\dots a_s} = 0 \qquad \Longleftrightarrow \qquad \delta\Phi_{\mu_1\dots\mu_s} = \nabla_{\mu_1}\xi_{\mu_2\dots\mu_s}$$

Free CFTs, e.g. free SYM or free scalar/fermion CFT, have ∞-many HS currents $\boldsymbol{J_s = \mathrm{Tr}[\phi\, \partial ... \partial\, \phi]}$, hence, the dual theory has to be a HiSGRA (Sundborg; Klebanov, Polyakov; Sezgin, Sundell; Leigh, Petkou; Jevicki et al; ...):



$$\sim \int_{AdS} \Phi^3 \qquad \Longleftrightarrow \qquad \langle JJJ \rangle =$$

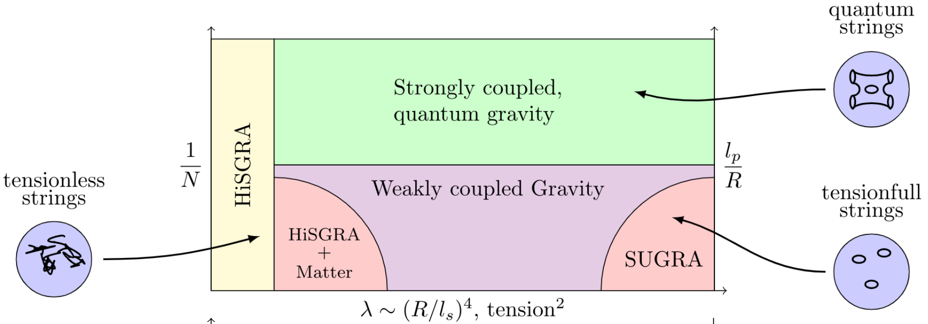


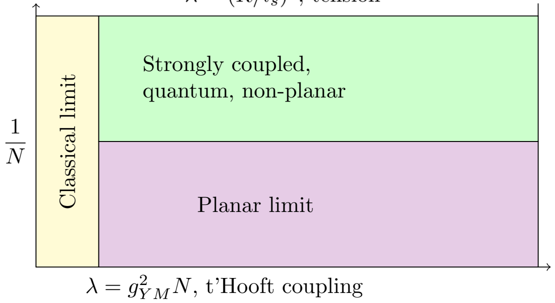
$$\neq 0$$

and **Gravity duals of even free CFT's are (highly) interacting theories.**

Weakly coupled CFT's do not have the large gap! Therefore, there is no well-defined semi-classical local gravitational dual by default (Heemskerk, Penedones, Polchinski, Sully; Maldacena, Simmons-Duffin, Zhiboedov)

HiSGRA from Tensionless Strings, duals of weakly coupled CFT's





Sundborg;
Klebanov, Polyakov;
Sezgin, Sundell;
Tseytlin;
Beisert, Bianchi, Morales, Samtleben;
Leigh, Petkou

Some 'tensionless pairs'/duals of weakly-coupled CFT's

- free SYM vs. ??HiSGRA+Matter (Sundborg; Sezgin, Sundell; Beisert, Bianchi, Morales, Samtleben);
- free SYM vs. tensionless strings on $AdS_5 \times S^5$ (Gaberdiel, Gopakumar);
- $\mathrm{Sym}^N(\mathbb{T}^4)$ vs. strings on $AdS_3 \times S^3 \times \mathbb{T}^4$ (Gaberdiel, Gopakumar; Eberhardt, Gaberdiel);
- Fishnet vs. ?? strings (Caetano, Kazakov; Gromov, Sever)
- free scalar/fermion CFT's, Wilson-Fisher/Gross-Neveu, Chern-Simons Matter theories up to ABJ vs. strings and ??HiSGRA (Klebanov, Polyakov, Sezgin, Sundell, Petkou, Leigh, Chang, Minwalla, Sharma, Yin, Giombi, Prakash, Trivedi, Wadia);
- ??subector of tensionless strings on $AdS_4 \times \mathbb{CP}^3$ vs. ??subsector of ABJ (Chern-Simons) vector models vs. Chiral HiSGRA (Ponomarev, E.S.; Sharapov, E.S., Van Dongen)

HiSGRA that survived

Quantizing Gravity via HiSGRA = Constructing Classical HiSGRA

Therefore, HiSGRA can be good probes of the Quantum Gravity Problem

Four classes of local HiSGRA in 2020

3d massless and partially-massless (Blencowe; Bergshoeff, Blencowe, Stelle; Campoleoni, Fredenhagen, Pfenninger, Theisen; Henneaux, Rey; Gaberdiel, Gopakumar; Grumiller et al; Grigoriev, Mkrtchyan, E.S.), $S = S_{CS}$ for a higher spin extension of $sl_2 \oplus sl_2$

$$S = \int \omega d\omega + \tfrac{2}{3}\omega^3$$

3d conformal (Pope, Townsend; Fradkin, Linetsky; Grigoriev, Lovrekovic, E.S.), $S = S_{CS}$ for higher spin extension of $so(3,2)$

4d conformal (Tseytlin, Segal; Bekaert, Joung, Mourad; Adamo, Tseytlin), higher spin extension of Weyl gravity, local Weyl symmetry tames non-localities

$$S = \int \sqrt{g}\,(C_{\mu\nu,\lambda\rho})^2 + ...$$

4d massless chiral (Metsaev; Ponomarev, E.S.; Ponomarev; E.S., Tran, Tsulaia; E.S.). The smallest higher spin theory with propagating fields. **This talk!**

The theories avoid all no-go's. Surprisingly, all of them have simple actions and are clearly well-defined, as close to Field Theory as possible

Other ideas and proposals

- **Reconstruction**: invert AdS/CFT
  - Brute force (Bekaert, Erdmenger, Ponomarev, Sleight; Taronna, Sleight)
  - Collective Dipole (Jevicki, Mello Koch et al; Aharony et al)
  - Holographic RG (Leigh et al, Polchinski et al)
- **IKKT matrix model for fuzzy H_4** (Steinacker, Sperling, Fredenhagen, Tran)
- **Formal HiSGRA**: constructing L_∞-extension of HS algebras, i.e. a certain odd Q, $QQ=0$, and write AKSZ sigma model (Barnich, Grigoriev)

$$d\Phi = Q(\Phi)$$

Warning: Boulanger, Kessel, E.S., Taronna

(Vasiliev; E.S., Sharapov, Bekaert, Grigoriev, E.S.; Grigoriev, E.S.; Tran; Bonezzi, Boulanger, Sezgin, Sundell; Neiman) AdS/CFT: (Sundborg, Sezgin, Sundell, Klebanov, Polyakov, Giombi, Yin, ...) Chiral HiSGRA (Sharapov, E.S., Sukhanov, Van Dongen)

Certain things do work, but the general rules are yet to be understood, e.g. non-locality, relation to field theory, quantization, ...

Chiral Higher Spin Gravity

Self-dual Yang-Mills is a useful analogy

- the theory is non-unitary due to the interactions ($A_\mu \rightarrow \Phi^\pm$)

$$\mathcal{L}_{\text{YM}} = \mathrm{tr}\, F_{\mu\nu}F^{\mu\nu}$$

$$\wr\wr$$

$$\mathcal{L}_{\text{SDYM}} = \Phi^- \Box \Phi^+ + V^{++-} + V^{--+} + V^{++--}$$

- tree-level amplitudes vanish, $A_{\text{tree}} = 0$
- one-loop amplitudes do not vanish and coincide with $(++\ldots+)$ of QCD
- **SD theories are consistent truncations**, so anything we can compute will be a legitimate observable in the full theory
- Yang-Mills Theory as a perturbation of SDYM is a fruitful idea
- integrability, instantons, twistors, ...

Chiral HSGRA in Minkowski

Chiral HiSGRA (Metsaev; Ponomarev, E.S.) is a 'higher spin extension' of SDYM/SDGR. It has fields of all spins $s = 0, 1, 2, 3, ...$:

$$\mathcal{L} = \sum_{\lambda} \Phi^{-\lambda} \Box \Phi^{+\lambda} + \sum_{\lambda_i} \frac{\kappa\, l_{\text{Pl}}^{\lambda_1+\lambda_2+\lambda_3-1}}{\Gamma(\lambda_1+\lambda_2+\lambda_3)} V^{\lambda_1,\lambda_2,\lambda_3}$$

light-cone gauge is very close to the spinor-helicity language

$$V^{\lambda_1,\lambda_2,\lambda_3} \sim [12]^{\lambda_1+\lambda_2-\lambda_3}[23]^{\lambda_2+\lambda_3-\lambda_1}[13]^{\lambda_1+\lambda_3-\lambda_2}$$

Locality + Lorentz invariance + genuine higher spin interaction result in a unique completion

This is the smallest higher spin theory and it is unique.
Graviton and scalar field belong to the same multiplet

No UV Divergences! One-loop finiteness

Tree amplitudes vanish. The interactions are naively non-renormalizable, the higher the spin the more derivatives:

$$V^{\lambda_1,\lambda_2,\lambda_3} \sim \partial^{|\lambda_1+\lambda_2+\lambda_3|}\Phi^3$$

but there are **no UV divergences!** (E.S., Tsulaia, Tran). Some loop momenta eventually factor out, just as in $\mathcal{N}=4$ SYM, but ∞-many times.

At one loop we find three factors: (1) SDYM or all-plus 1-loop QCD; (2) higher spin dressing to account for λ_i; (3) total number of d.o.f.:

$$A^{\text{1-loop}}_{\text{Chiral}} = A^{++\ldots+}_{\text{QCD,1-loop}} \times D^{\text{HSG}}_{\lambda_1,\ldots,\lambda_n} \times \sum_\lambda 1$$

$\#$ d.o.f.$= \sum_\lambda 1 = 1 + 2\sum_{\lambda>0} 1 = 1 + 2\zeta(0) = 0$ to comply with no-go's, (Beccaria, Tseytlin) and agrees with many results in AdS, where $\neq 0$

Chiral HSGRA in Minkowski

- **stringy 1**: the spectrum is infinite $s = 0, (1), 2, (3), 4, ...$
- **stringy 2**: admit Chan-Paton factors, $U(N)$, $O(N)$ and $USp(N)$
- **stringy 3**: we have to deal with spin sums $\sum_s$ (worldsheet takes care of this in string theory) and ζ-function helps
- **stringy 4**: the action contains parts of YM and Gravity
- **stringy 5**: higher spin fields soften amplitudes
- consistent with Weinberg etc. $S = 1^{***}$ (in Minkowski)
- gives all-plus QCD or SDYM amplitudes from a gravity

Apart from Minkowski space the theory exists also in (anti)-de Sitter space, where holographic S-matrix turns out to be nontrivial ... and related to Chern-Simons matter theories

Covariantization, SDYM, SDGR

Chiral HiSGRA admits two contractions (Ponomarev) to higher spin extensions of SDYM and SDGR. These HS-SDYM and HS-SDGR can be covariantized (Krasnov, E.S., Tran). New (Hitchin) free action

$$\boldsymbol{S = \int \Psi^{A_1\dots A_{2s}} \wedge H_{A_1A_2} \wedge \nabla\omega_{A_3\dots A_{2s-2}}}$$

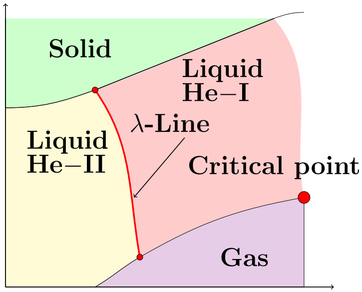
$H^{AB} \equiv e^A{}_{C'} \wedge e^{BC'}$. Interactions can be introduced by taking sum over s and by replacing $\nabla\omega$ or both H and $\nabla\omega$ with

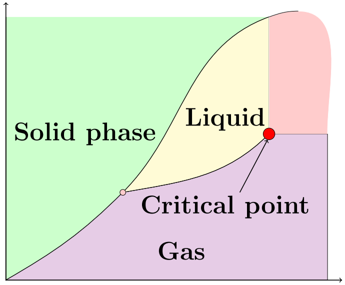
$$F = d\omega + \tfrac{1}{2}[\omega, \omega]$$

where $\omega \equiv \sum_k \omega_{A_1\dots A_k}\, y^{A_1}\dots y^{A_k}$ and the commutator is either due to Yang-Mills groups or due to Poisson bracket on $\mathbb{R}^2$ of $f(y)$, same as $w_{1+\infty}$.

Full covariant form of Chiral HiSGRA is available (Sharapov, E.S., Sukhanov, Van Dongen). Twistors (Tran)

Chern-Simons Matter Theories and bosonization duality





Chern-Simons Matter theories and dualities

In AdS_4/CFT_3 one can do much better — there exists a large class of models, Chern-Simons Matter theories (extends to ABJ(M))

$$\frac{k}{4\pi}S_{CS}(A) + \text{Matter}\begin{cases}(D\phi^i)^2 & \textbf{free boson}\\ (D\phi^i)^2 + g(\phi^i\phi^i)^2 & \textbf{Wilson-Fisher (Ising)}\\ \bar{\psi}\not{D}\psi & \textbf{free fermion}\\ \bar{\psi}\not{D}\psi + g(\bar{\psi}\psi)^2 & \textbf{Gross-Neveu}\end{cases}$$

- describe physics (Ising, quantum Hall, ...)
- break parity in general (Chern-Simons)
- two parameters $\lambda = N/k$, $1/N$ (λ continuous for N large)
- exhibit remarkable dualities, e.g. ***3d* bosonization duality** (Aharony, Alday, Bissi, Giombi, Karch, Maldacena, Minwalla, Prakash, Seiberg, Tong, Witten, Yacobi, Yin, Zhiboedov, ...)

Chern-Simons Matter theories and dualities

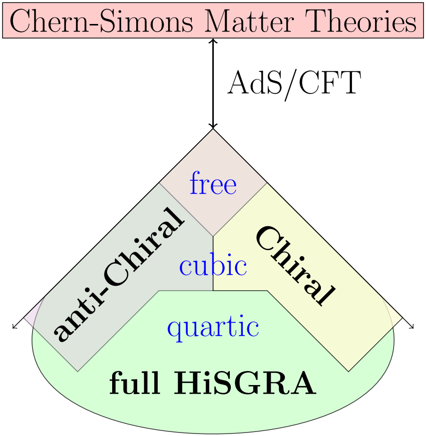
$$\begin{tikzcd}[column sep=huge, row sep=huge]
\textbf{free boson} \arrow[r, shift left, "\tilde{\lambda}_b\text{-coupling}"] \arrow[d, dash, "\text{RG flow}"'] & \textbf{Gross-Neveu} \arrow[l, shift left, "\tilde{\lambda}_f\text{-coupling}"] \arrow[d, dash, "\text{RG flow}"] \\
\textbf{Wilson-Fisher} \arrow[r, shift left, "\tilde{\lambda}_b\text{-coupling}"] & \textbf{free fermion} \arrow[l, shift left, "\tilde{\lambda}_f\text{-coupling}"]
\end{tikzcd}$$

The simplest gauge-invariant operators are $J_s = \phi D ... D\phi$ or $J_s = \bar{\psi}\gamma D...D\psi$, which are dual to higher spin fields.

Currents are slightly non-conserved $\partial \cdot J = \frac{1}{N}[JJ]$

$\gamma(J_s)$ at order $1/N$ (Giombi, Gurucharan, Kirillin, Prakash, E.S.) confirm the duality. Many other tests!

Chiral HiSGRA and Chern-Simons Matter



$\exists$ Chiral HiSGRA $\rightarrow$ $\exists$ closed subsector

(anti)-Chiral Theories are rigid, we need to learn how to glue them

gluing depends on one parameter, which is introduced via simple EM-duality rotation $\Phi_{\pm s} \rightarrow e^{\pm i\theta}\Phi_{\pm s}$

gives all 3-point correlators consistent with (Maldacena, Zhiboedov)

Bosonization is manifest! Concrete predictions from HiSGRA.

(anti)-Chiral Theories provide a complete base for 3-pt amplitudes

$$\boldsymbol{V_3 = V_{chiral} \oplus \bar{V}_{chiral} \quad \leftrightarrow \quad \langle JJJ \rangle}$$

Higher spin symmetry and bosonization duality

In free theories we have ∞-many conserved $J_s = \phi\partial...\partial\phi$ tensors.

Free CFT = Associative (higher spin) algebra

Conserved tensor $\rightarrow$ current $\rightarrow$ symmetry $\rightarrow$ invariants=correlators.

$$\partial \cdot J_s = 0 \quad \Longrightarrow \quad Q_s = \int J_s \quad \Longrightarrow \quad [Q, Q] = Q \quad \& \quad [Q, J] = J$$

HS-algebra (free boson) = HS-algebra (free fermion) in $3d$.

Correlators are given by invariants (Sundell, Colombo; Didenko, E.S.; ...)

$$\langle J...J\rangle = \mathrm{Tr}(\Psi \star ... \star \Psi) \qquad \Psi \leftrightarrow J \qquad \boldsymbol{\Psi \star \Psi = \Psi}$$

where Ψ are coherent states representing J in the higher spin algebra

$$\langle JJJJ\rangle_{F.B.} \sim \cos(Q^2_{13} + Q^3_{24} + Q^4_{31} + Q^1_{43}) \cos(P_{12}) \cos(P_{23}) \cos(P_{34}) \cos(P_{41}) + ...$$

Slightly-broken Higher spin symmetry is new Virasoro?

In large-N Chern-Simons vector models (e.g. Ising) higher spin symmetry does not disappear completely (Maldacena, Zhiboedov):

$$\partial \cdot J = \frac{1}{N}[JJ] \qquad\qquad [Q, J] = J + \frac{1}{N}[JJ]$$

What is the right math? We should deform the algebra together with its action on the module, so that the currents can 'backreact':

$$\delta_\xi J = l_2(\xi, J) + l_3(\xi, J, J) + \dots\,, \qquad\qquad [\delta_{\xi_1}, \delta_{\xi_2}] = \delta_\xi\,,$$

where $\xi = l_2(\xi_1, \xi_2) + l_3(\xi_1, \xi_2, J) + \dots$. This leads to L_∞-algebra.

Correlators = invariants of L_∞-algebra and are unique (Gerasimenko, Sharapov, E.S.), **which proves $3d$ bosonization duality at least in the large-N**. Without having to compute anything one prediction is

$$\langle J \dots J \rangle = \sum \langle \text{fixed} \rangle_i \times \text{params}$$

Massive Higher Spins

(no no-go's, no challenge?)

Massive higher spins?

... but string's spectrum is full of massive higher spins ...

Massive higher spins are notoriously more complicated: second class constraints, Boulware-Deser ghosts, actions are not easy (Singh, Hagen; Zinoviev)

$$(\Box - m^2)\Phi_{\mu_1\dots\mu_s} = 0 \qquad\qquad \partial^\nu \Phi_{\nu\mu_2\dots\mu_s} = 0$$

Low spins: $s = 1$ spontaneously broken Yang-Mills; $\boldsymbol{s = 3/2}$; $s = 2$ massive (bi)-gravity (dRGT; Hassan, Rosen)

Simple idea in $4d$ (Ochirov, E.S.): instead of $\Phi_{A(s),A'(s)}$, i.e. (s,s) of $sl(2,\mathbb{C})$ we suggest chiral description $\Phi_{A_1\dots A_{2s}}$, i.e. $(2s,0)$. Parity is not easy ...

Easy to introduce EM, YM and gravitational interactions, all-helicity-plus amplitudes are reproduced; relation to black-hole scattering (Arkani-Hamed, Huang^2; Guevara, Ochirov, Vines). Of course, these are effective field theories ... but everything small and rotating is a higher spin particle from a distance

Random comments

Random comments

HiSGRA in dS as well: prediction for R vs. R^3 corrections in cosmology, (Anninos et al,...)

Quantum checks in conformal HiSGRA and holographic (hands free)

Holographic HiSGRA's amplitudes seem to be closer to Veneziano's

Higher spin algebra = deformation quantization of coadjoint orbits = uirrep of free conformal fields (AdS/CFT built in); Moyal-Weyl in the simplest cases

Holographic correlators: $\Psi \star \Psi = \Psi$ in Moyal-Weyl = Wick contractions

$d\Phi = Q(\Phi)$ gives examples of L_∞'s that originate from A_∞; turn out to be fixed by DQ of the associated Poisson orbifold; sometimes there is HPT

Many formal exact solutions ... also projectors

Observables, etc.: Chevalley-Eilenberg, cyclic and Hochschild cohomology

Summary

- Some HiSGRA do exist as local field theories, e.g. Chiral HSGRA — toy model with stringy features. Some quantum checks passed **no UV divergences, supersymmetry vs. higher spin symmetry**. Relation to Chern-Simons vector models and $3d$ bosonization.
- Many AdS/CFT pairs with (non-local) HiSGRA on one end and string theory on another; stringy features of HiSGRA ...
- (Slightly-broken) higher symmetry is interesting on its own, L_∞ as physical symmetry
- Higher structures: homological perturbation theory, A_∞, L_∞
- Higher spins and SFT should have a round table!

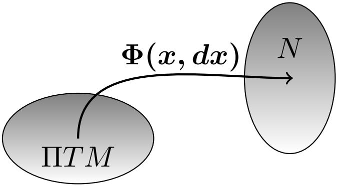
Thank you for your attention!

may the higher spin force be with you!

... backup slides ...

Let us be given a Q-manifold (view it locally as an L_∞-algebra)

$\Omega^\bullet(M)\,, d^2=0$



$\Phi\,, Q^2=0$

then we can always write a sigma-model:

$$\boldsymbol{d\Phi = Q(\Phi)}$$

Any PDE can be cast into such a form ... (Barnich, Grigoriev)

Other names: Free Differential Algebras (Sullivan), in physics: (van Nieuwenhuizen; Fre, D' Auria); FDA=unfolding (Vasiliev), AKSZ (AKSZ)

Any PDE can be cast into such a form ... but in practice cannot

Einstein equations are already painful enough

$$
\begin{aligned}
de^a &= \omega^{a,}{}_b \wedge e^b && \text{no torsion}\\
d\omega^{a,b} &= \omega^{a,}{}_c \wedge \omega^{c,b} + e_m \wedge e_n C^{ab,mn} && \text{Einstein is here!}\\
dC^{ab,mn} &= \omega C^{ab,cd} + e_k \boldsymbol{C^{ab,mn,k}} && \text{Bianchi for Weyl}
\end{aligned}
$$

vielbein $e^a \equiv e^a_\mu \, dx^\mu$; spin-connection $\omega^{a,b} \equiv \omega^{a,b}_\mu \, dx^\mu$; Weyl tensor $C^{ab,cd}$; **and more $C^{ab,cd,k}$, ... and nonlinear equations for them**

We have to introduce auxiliary fields to parameterize the on-shell jet:

$$\boldsymbol{R_{ab,mn}} = C_{ab,mn} \qquad (\textbf{Riemann} = \text{its Weyl}) \sim \text{Einstein}$$

Closed-form for SDYM and SDGR (E.S., Van Dongen), l_2, l_3

Higher Spin Gravity as Quantum Gravity?

(quantum tests in the not-yet-existing theories)

Quantum Effects in Holographic HiSGRA

Given that there are very few HiSGRA's with action and propagating massless fields, while holographic HiSGRA are very non-local, people have to be very creative to say anything about quantum corrections:

- AdS/CFT-inspired: use one-loop determinants to probe the spectrum on various backgrounds;
- AdS/CFT-inspired: use AdS unitarity cuts as to reduce calculation of loops to manipulation with CFT data;
- explicit computations in Chiral HiSGRA
- for de-Sitter space see (Anninos, Denef, Law, Sun)

One-loop determinants in Minkowski HiSGRA

Warming up: HiSGRA can look like topological theories. Indeed, suppose we are in Minkowski space

$$S = \sum_s \int \Phi_{a_1\ldots a_s} \Box\, \Phi^{a_1\ldots a_s} \qquad\qquad \Phi_{a_1\ldots a_s} = \partial_{a_1} \xi_{a_2\ldots a_s}$$

The one-loop partition function is

$$Z^2 = \frac{1}{\det_{s=0}\Box} \frac{\det_{s=0}\Box \det_{s=1}\Box \ldots}{\det_{s=1}\Box \det_{s=2}\Box \ldots} = \frac{1}{(\det_{s=0}\Box)^{\nu_{eff}}} \overset{?}{=} 1$$

Beccaria, Tseytlin suggested to cancel (de)numerators, so that

$$\nu_{eff} = \sum_\lambda 1 = 1 + \sum_{s>0} 2 = 1 + 2\zeta(0) = 1 - 1 = 0$$

One-loop determinants in Minkowski HiSGRA

In AdS a naive cancellation is not possible

$$S=\sum_s \int \Phi_s \mathrm{K}_s \Phi_s\,, \quad \mathrm{K}_s=-\nabla^2+M_s^2\,, \quad \delta\Phi_{a_1\dots a_s}=\nabla_{a_1}\xi_{a_2\dots a_s}$$

The one-loop partition function is

$$e^{-2\boldsymbol{F^1_{AdS}}}=Z^2=\frac{1}{\det \mathrm{K}_0}\frac{\det \tilde{\mathrm{K}}_0 \det \tilde{\mathrm{K}}_1 \dots}{\det \mathrm{K}_1 \det \mathrm{K}_2 \dots}=?$$

We also have predictions from AdS/CFT: $Z_{CFT}=Z_{AdS}$

$$F_{AdS}=\frac{1}{G}F^0_{AdS}+\boldsymbol{F^1_{AdS}}+\mathcal{O}(G)$$

$$F_{CFT}=N\boldsymbol{F^0_{CFT}}+F^1_{CFT}+\mathcal{O}(\frac{1}{N})$$

For example, the free scalar in $3d$ gives $\boldsymbol{F^3_\phi}=\frac{1}{16}(2\log 2-\frac{3\zeta(3)}{\pi^2})$

One-loop determinants in Minkowski HiSGRA

It seems that we should be proving $0 = 0$, but this is not the case!

- Different backgrounds: Euclidian, global, thermal AdS_{d+1} spaces etc. allow us to get an access to a, c anomaly coefficients, Casimir energy, sphere free energy
- Different spectrum of fields, e.g. all spins or even only vs. $U(N)$ or $O(N)$ CFT duals
- There can be various $\log$-divergences that should either agree or cancel

$$F_{AdS,s} = -\zeta_s(0)\log\Lambda l - \frac{1}{2}\zeta'_s(0)$$

ζ's for various fields were found by Camporesi, Higuchi, but we need to **sum over infinitely many fields, let's try zeta-function!**

An interesting pattern observed for a number of low dimensions d: (Giombi, Klebanov, Safdi, Tseytlin, Beccaria, ...)

- all integer spins: $\boldsymbol{F^1_{AdS}} = 0$, ok we have $0 = 0$;
- for even spins: $\boldsymbol{F^1_{AdS}} = \boldsymbol{F^0_{CFT}}$, so the duality can work if $G^{-1} = N - 1$ and we have

$$\begin{aligned} F_{AdS} &= (N-1)\boldsymbol{F^0_{CFT}} + \boldsymbol{F^0_{CFT}} + \mathcal{O}(G) \\ F_{CFT} &= (N-1+1)\boldsymbol{F^0_{CFT}} \end{aligned}$$

Similar shifts of G are in Chern-Simons dual to topological strings (Sinha, Vafa) and for the usual SYM vs. IIB on $AdS_5 \times S^5$ duality;

It is hard to 'fake' $F^3_\phi = \frac{1}{16}(2\log 2 - \frac{3\zeta(3)}{\pi^2})$ and similar numbers

One-loop determinants in Minkowski HiSGRA

There is a general analytical proof of the conjecture (E.S, Tung) and its extension to non-integer dimension (Klebanov, Polyakov), $AdS_{4.99}/CFT^{3.99}$

In particular, in AdS_4/CFT^3 for Wilson-Fisher

$$\delta\tilde{F} = -\frac{\zeta(3)}{8\pi^2}$$

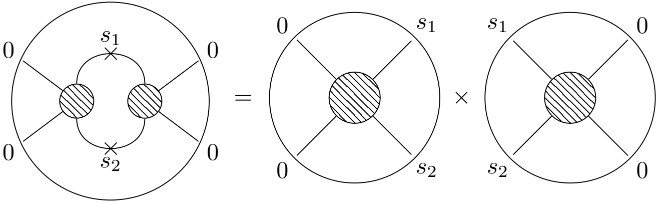
In particular, around $AdS_{4.99}/CFT^{3.99}$ for Wilson-Fisher

$$\delta\tilde{F} = -\frac{\pi}{567}\epsilon^3 - \frac{13\pi}{6912}\epsilon^4 + \ldots$$

The ϵ-expansion of free energy of the Wilson-Fisher CFT is reproduced as one-loop effect in Higher Spin Gravity

Unitarity cuts in AdS/CFT (non-invasive techniques)

One-loop four-point can be reconstructed (Ponomarev, Sezgin, E.S.)



from its double-cut following Fitzpatrick, Kaplan.

- the double-cut turns out to be higher spin invariant
- agreement with $G^{-1} \sim N$ or $G^{-1} \sim N \pm 1$

Thank you for your attention again!