# String worldsheet models of black hole microstates

David Turton

University of Southampton

String Field Theory 2022, Prague

16 Sep 2022

Based on:

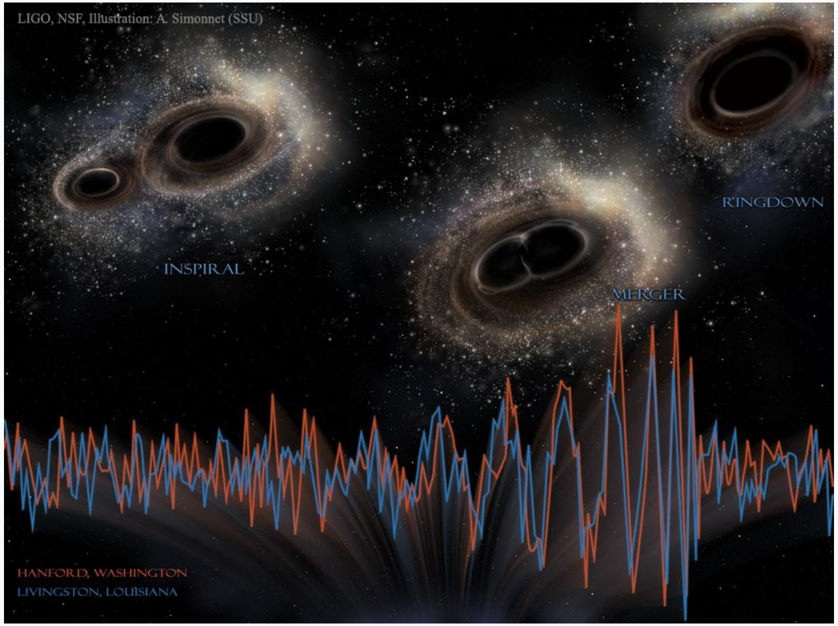
Martinec, Massai, DT ’19, ’20
Bufalini, Iguri, Kovensky, DT ’21 & 2203.13828, PRL

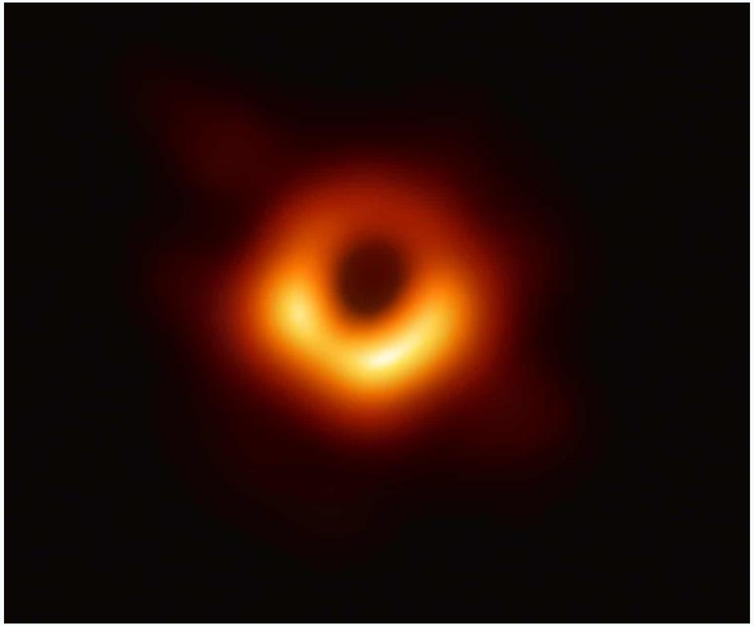
UNIVERSITY OF Southampton

STAG RESEARCH CENTRE

THE ROYAL SOCIETY

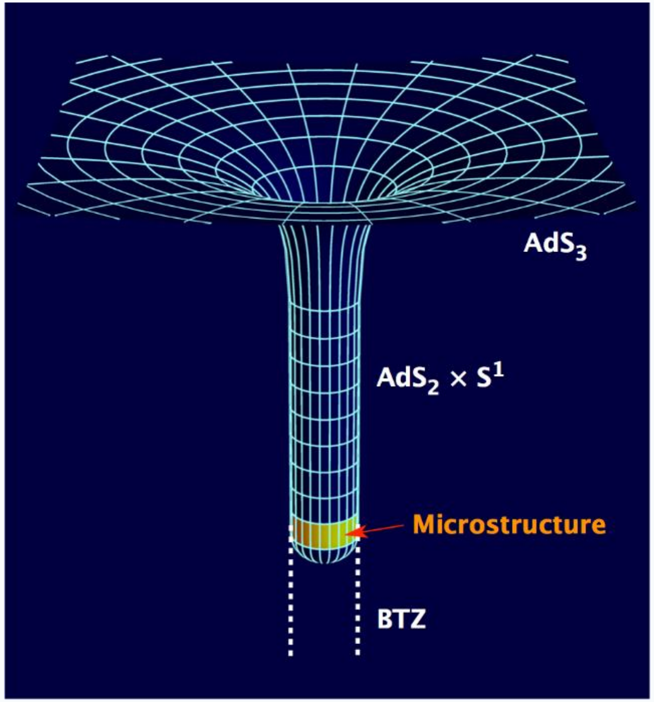
We live in exciting times, and the time is ripe to further the exploration of the internal microstructure of black holes.



Much progress has been made in describing pure black hole microstates using smooth horizonless supergravity solutions.

In AdS, these are holographically dual to heavy pure CFT states, and are interpreted as (coherent, pure) black hole microstates.




Such black hole microstate geometries provide examples of string-theoretic quantum structure on the scale of the black hole event horizon.

However there are features

of black hole microstates that

require us to go beyond such

smooth supergravity solutions.

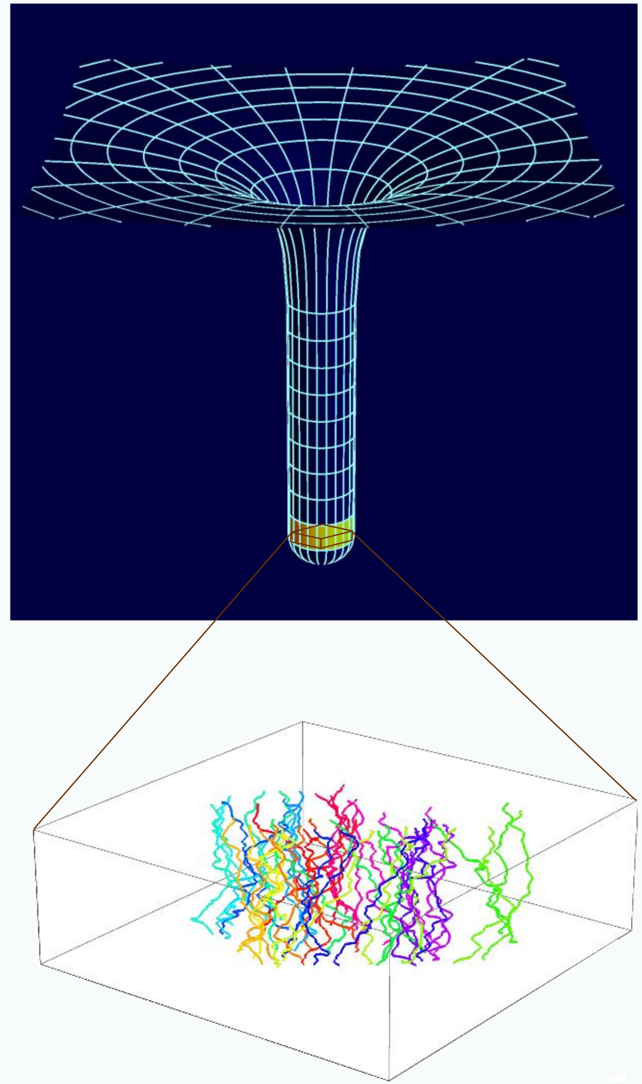
These features include the fine

microstructure of the bound state,

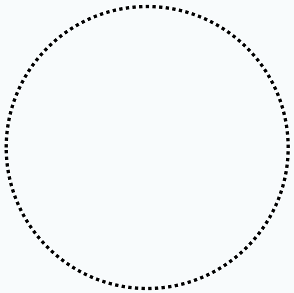
as well as the spectrum and

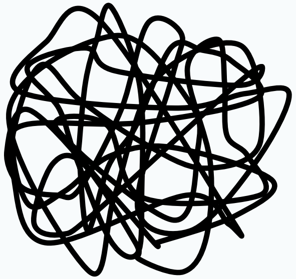
dynamics of probe fundamental

strings and D-branes.



Black hole microstate geometries and their stringy counterparts are interpreted in the context of the broader fuzzball proposal, that **quantum** effects are important at the scale of the would-be event horizon, due to the **finite size** of the underlying quantum bound state, such that Hawking radiation is unitary. This would resolve the black hole information paradox.





# Outline

1. Intro and review
2. Worldsheet models for black hole microstates
3. Heavy-light correlators from the worldsheet
4. D-brane probes and general gauged models

# Two-charge Black hole

Consider a multi-wound fundamental string F1 carrying momentum P.

- Entropy: exponential degeneracy of microscopic states

Sen '94

- For classical profiles, string sources good supergravity background

  Classical profiles $\leftrightarrow$ coherent states

Dabholkar, Gauntlett, Harvey, Waldram '95
Lunin, Mathur '01

- No horizons; string source

- Transverse vibrations only $\rightarrow$ non-trivial size

F1-P is U-dual to F1-NS5 or D1-D5 bound state

- Configurations are everywhere smooth in these duality frames

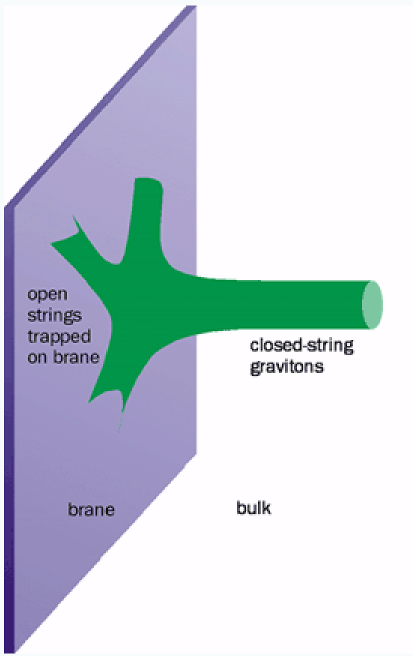
- Can study precision holography.

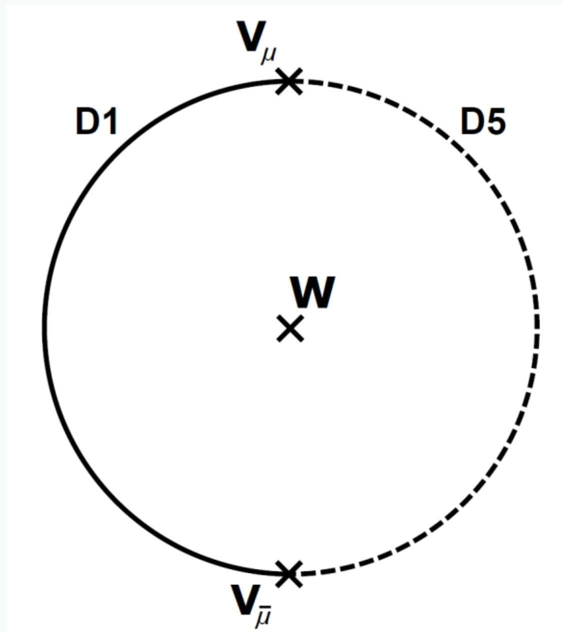
Skenderis, Taylor '05–'08
Giusto, Moscato, Russo '15
(Giusto), Rawash, Turton '19, '21

# Backreaction from boundary state

Worldsheet string theory can give insight into such heavy bound states.

For instance, using open-closed disk amplitudes, one can derive the asymptotic supergravity fields sourced by the branes, in a 1/r expansion.




Black, Russo, DT '10,
Giusto, Russo, DT '11

These open-closed disk amplitudes provided important input into the supergravity microstate program, by demonstrating that D1-D5-P bound states source more fields than were under active consideration at the time.

(For configurations invariant on a compact $T^4$, one obtains 6D minimal supergravity coupled to **two** tensor multiplets.)

These open-closed disk amplitudes provided important input into the supergravity microstate program, by demonstrating that D1-D5-P bound states source more fields than were under active consideration at the time.

(For configurations invariant on a compact $T^4$, one obtains 6D minimal supergravity coupled to **two** tensor multiplets.)

However this approach only describes the region **far from** the branes.

Today I will report on a different approach that gives access to the stringy microstructure **deep inside** the near-brane region.

We shall work with NS5-F1-P bound states, with backgrounds that have pure NS-NS flux, and use the RNS formalism with BRST quantization.

# Stringy structure of black hole microstates

String theory contains much more than supergravity.

To what extent is the physics of strings and branes necessary to describe black hole interior structure?

On general grounds, may be expected to be important.

Example: Microstate geometries contain topological cycles at the bottom of a throat; branes wrapping those cycles are massive, but become light as one increases the length of the throat. Known as “W-branes”.

# NS5-F1-P System

Consider Type IIB string theory compactified on $S^1 \times M_4$ where $M_4 = T^4$ or K3 (take $T^4$ for concreteness).

- Radius of $S^1$ : $R_y$
- $n_5$ NS5 branes wrapped on $S^1 \times T^4$
- $n_1$ units of fundamental string winding on $S^1$
- $n_P$ units of momentum along $S^1$

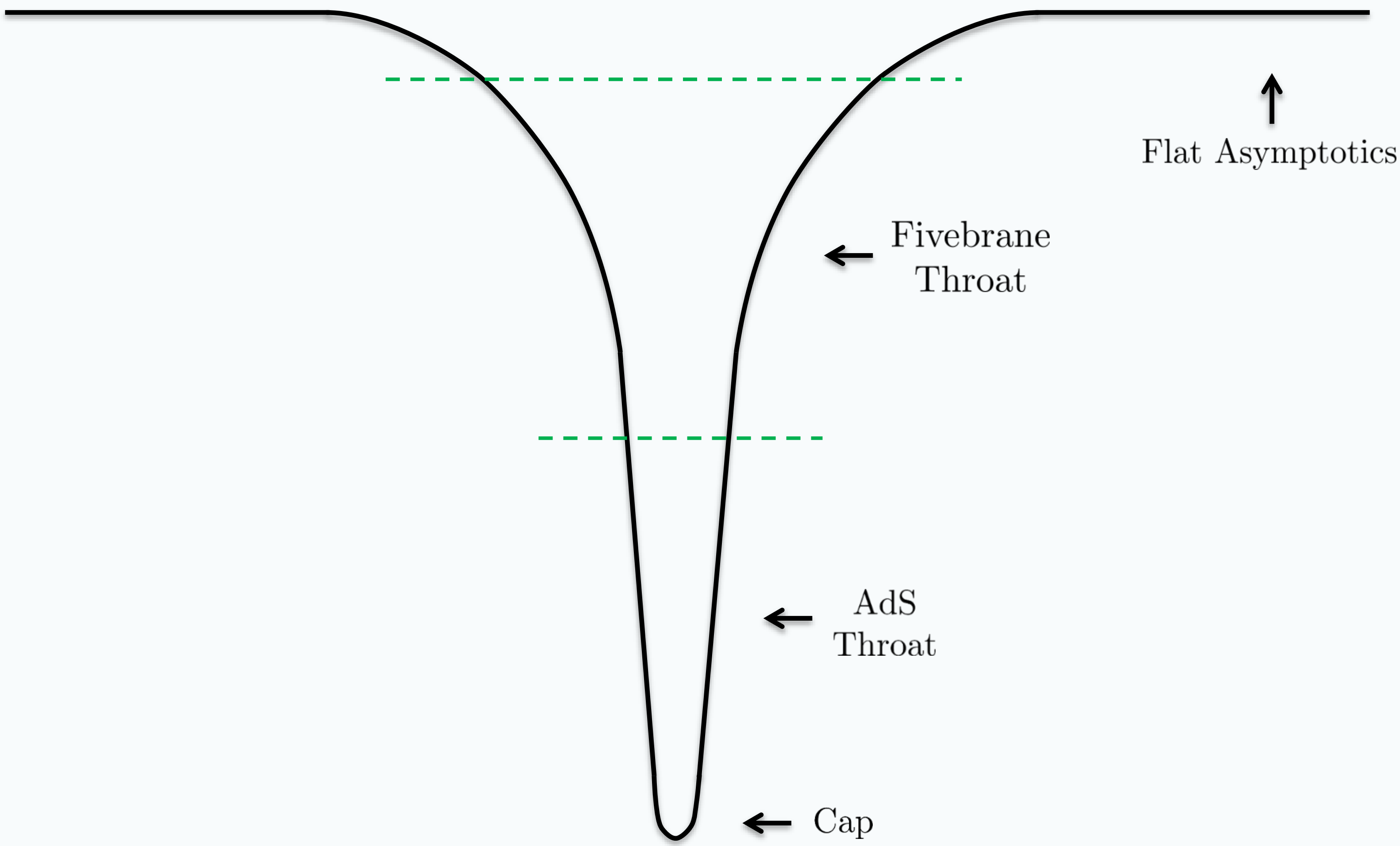
For states which have geometrical descriptions, the geometry has charges

$$Q_1 = \frac{g_s^2 \alpha'^3}{V} n_1 , \qquad Q_5 = \alpha' n_5 , \qquad Q_P = \frac{g_s^2 \alpha'^4}{V R_y^2} n_P .$$

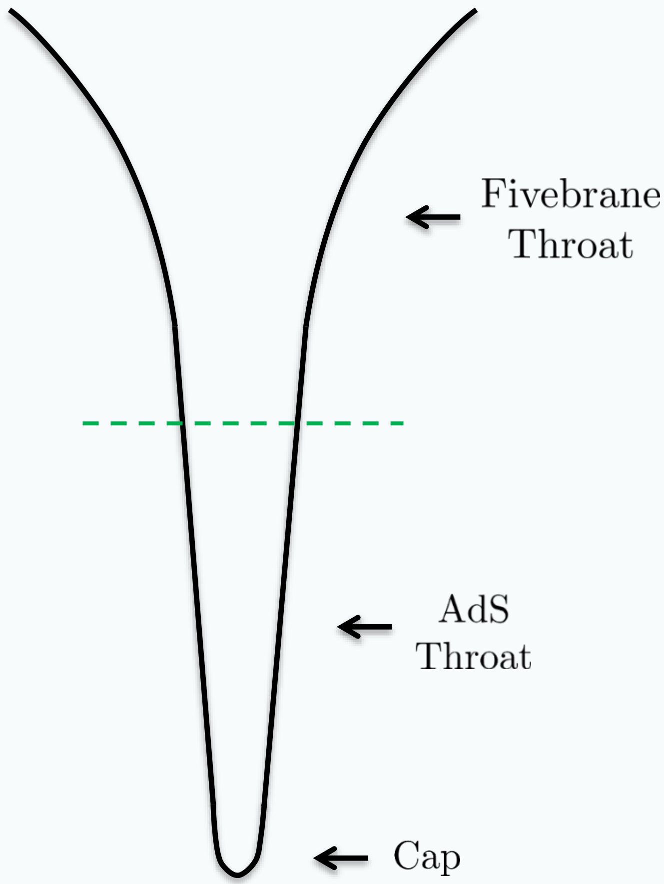
We work in the large $R_y$ supergravity regime, in which we have the hierarchy of scales

$$Q_5 \gg Q_1 \gg Q_p , \qquad \frac{Q_5}{\alpha'} \equiv n_5 \gg 1 .$$

We work in the NS5-brane decoupling limit:

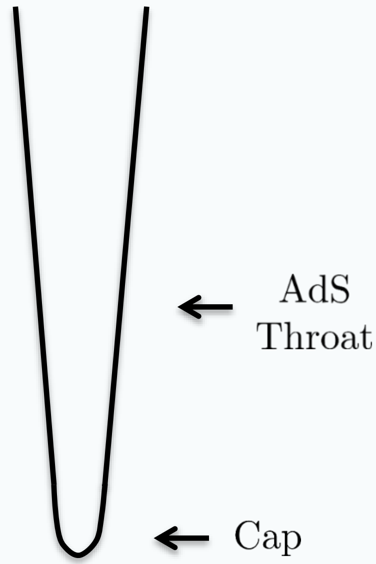


We work in the NS5-brane decoupling limit:



We will also work frequently in the AdS$_3$ decoupling limit,

in which we can study AdS$_3$/CFT$_2$ holographic duality.



# Symmetric product orbifold CFT

- Orbifold CFT on $(\mathrm{T}^4)^N/\mathrm{S}_N$ : N copies of $c=6$ $\mathrm{T}^4$ sigma model.

  Each copy has 4 bosons X , & 4 chiral + 4 antichiral fermions $(\psi,\ \bar{\psi})$.

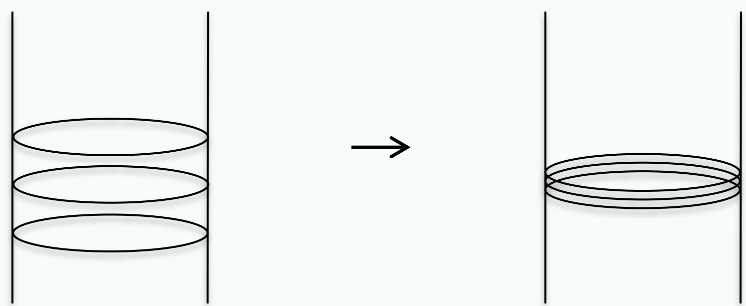
- **Twist operators**: permute fields, 'link together' different copies:

$$\sigma_{\mathsf{k}}: \qquad \begin{array}{l} X^{(1)} \to X^{(2)} \to \cdots \to X^{(\mathsf{k})} \to X^{(1)} \\ \psi^{(1)} \to \psi^{(2)} \to \cdots \to \psi^{(\mathsf{k})} \to \ -\psi^{(1)}. \end{array}$$

c.f. Pronobesh Maity's talk

- The operator $\sigma_{\mathsf{k}}$ links together k copies of the sigma model to effectively make a single CFT on a circle k times longer.

$\sigma_3$ :



'**Strand**' of length k

E.g. single-trace operator, corresponding to a supergravity state:

$$O^{++} = \sum_{r=1}^{N} O^{++}_{(r)} = \frac{1}{\sqrt{2}} \sum_{r=1}^{N} \psi^{+\dot{A}}_{(r)} \varepsilon_{\dot{A}\dot{B}} \bar{\psi}^{+\dot{B}}_{(r)} \,.$$

We shall consider this operator when demonstrating an explicit example worldsheet correlator shortly.

# Worldsheet models of black hole microstates

A particularly interesting family of black hole microstates arise from spectral flow of a family of backgrounds known as circular supertubes.

This family includes both supersymmetric and non-supersymmetric backgrounds.

Lunin;  Giusto, Mathur, Saxena ’04
Jejjala, Madden, Ross, Titchener ’05
Giusto, Lunin, Mathur, DT  ’12
Chakrabarty, Virmani, DT ’15

The worldsheet description of the spectral flowed supertubes is a particular gauged supersymmetric Wess-Zumino-Witten model,

$$\mathcal{S}_{WZW}(g,\mathsf{k}) = \frac{\mathsf{k}}{2\pi}\int \mathrm{Tr}\left[(\partial g)g^{-1}(\bar{\partial} g)g^{-1}\right] + \Gamma_{WZ}(g)\,.$$

WZW model is (10+2)-dimensional a priori – null gauging removes 1+1 directions,

$$\mathcal{G}/\mathcal{H} \quad = \quad \frac{\mathrm{SL}(2,\mathbb{R})_{n_5}\times \mathrm{SU}(2)_{n_5}\times \mathbb{R}_t \times \mathrm{S}^1_y}{\mathrm{U}(1)_\mathrm{L}\times \mathrm{U}(1)_\mathrm{R}} \times \mathrm{T}^4$$

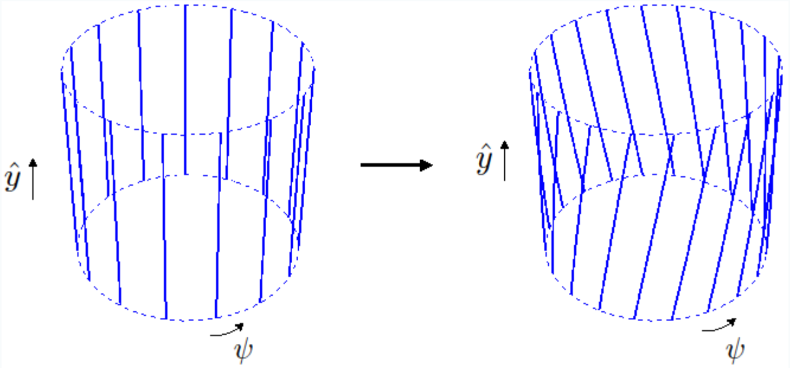
Asymmetric null gauging; null currents $\mathcal{J}\,,\bar{\mathcal{J}}$

$$\mathcal{S}^{\mathcal{G}}_{gWZW} \;=\; \mathcal{S}^{\mathcal{G}}_{WZW} + \frac{1}{\pi}\int d^2\hat{z}\left[\mathcal{A}\bar{\mathcal{J}} + \bar{\mathcal{A}}\mathcal{J} - \frac{\Sigma}{2}\bar{\mathcal{A}}\mathcal{A}\right]$$

Integrating out the gauge fields results in a sigma model on various backgrounds of interest, depending on the choice of null currents $\mathcal{J}\,,\bar{\mathcal{J}}$

Backgrounds that can be generated in this way include:

- NS5 branes on Coulomb branch, in a circular, symmetric configuration
- NS5-P helical supertube
- NS5-F1 helical supertube
- NS5-F1-P spectral flowed BPS supertubes
- NS5-F1-P JMaRT – spectral flowed non-BPS supertubes.




Martinec, Massai ’17

General models in this class: gauge null currents

$$\begin{aligned} \mathrm{U}(1)_{\mathrm{L}}: &\quad \mathcal{J} = l_1 J_3^{\mathrm{sl}} + l_2 J_3^{\mathrm{su}} + l_3 \partial t + l_4 \partial y\,, \\ \mathrm{U}(1)_{\mathrm{R}}: &\quad \bar{\mathcal{J}} = r_1 \bar{J}_3^{\mathrm{sl}} + r_2 \bar{J}_3^{\mathrm{su}} + r_3 \bar{\partial} t + r_4 \bar{\partial} y\,, \end{aligned}$$

where

$$0 = \langle \mathbf{l}, \mathbf{l} \rangle = n_5(-l_1^2 + l_2^2) - l_3^2 + l_4^2 \quad , \qquad 0 = \langle \mathbf{r}, \mathbf{r} \rangle = n_5(-r_1^2 + r_2^2) - r_3^2 + r_4^2$$

For convenience we set $l_1 = 1$.

Consistency of worldsheet spectrum $\leftrightarrow$ smoothness, absence of CTCs, & absence of horizons in sugra

Constrains the parameters to:

$$\begin{aligned}
l_2 &= \mathsf{m} + \mathsf{n} \in 2\mathbb{Z}+1\,, \qquad r_2 = -(\mathsf{m}-\mathsf{n}) \in 2\mathbb{Z}+1\,, \qquad \mathsf{m},\mathsf{n} \in \mathbb{Z}\,,\\
l_4 &= -\left(\mathsf{k}R_y - \frac{\mathsf{p}}{R_y}\right), \qquad r_4 = \mathsf{k}R_y + \frac{\mathsf{p}}{R_y} \qquad \mathsf{k},\mathsf{p} \in \mathbb{Z}\,,\\
l_3 &= r_3 = -\sqrt{\mathsf{k}^2 R_y^2 + \frac{\mathsf{p}^2}{R_y^2} + n_5\left(\mathsf{m}^2+\mathsf{n}^2-1\right)}\,,\\
\mathsf{k}\,\mathsf{p} &= n_5\,\mathsf{m}\,\mathsf{n}\,.
\end{aligned}$$

These parameters uniquely give the general spectral flowed supertubes.

$\rightarrow$ Integers k, m, n, $n_5$, $n_1$ and the scale $R_y$.

Bufalini, Iguri, Kovensky, DT '21

$$
\begin{aligned}
ds^2 \;&= n_5(d\theta^2 + d\rho^2) + \frac{1}{\Sigma_0}\bigg[ -\left(\sinh^2\!\rho + (\mathsf{m}^2 - \mathsf{n}^2)\cos^2\theta + 1 - \mathsf{m}^2 - \frac{\mathsf{p}^2}{n_5 R_y^2}\right) dt^2 \\
&\quad + \left(\sinh^2\!\rho + (\mathsf{m}^2 - \mathsf{n}^2)\cos^2\theta + \mathsf{n}^2 + \frac{\mathsf{p}^2}{n_5 R_y^2}\right) dy^2 - 2\,\frac{\mathsf{p}}{n_5 R_y}\,\Delta\, dt dy \\
&\quad + \left(n_5 \sinh^2\!\rho + n_5 \mathsf{m}^2 + \mathsf{k}^2 R_y^2\right) \sin^2\theta\, d\phi^2 + \left(n_5 \sinh^2\!\rho + n_5 \mathsf{n}^2 + \mathsf{k}^2 R_y^2\right) \cos^2\theta\, d\psi^2 \\
&\quad + 2\left(\mathsf{m}\Delta dt - \left(\mathsf{m}\frac{\mathsf{p}}{R_y} + \mathsf{n}\mathsf{k}R_y\right) dy\right) \sin^2\theta\, d\phi - 2\left(\mathsf{n}\Delta dt - \left(\mathsf{n}\frac{\mathsf{p}}{R_y} + \mathsf{m}\mathsf{k}R_y\right) dy\right) \cos^2\theta\, d\psi \bigg], \\
B \;&= \frac{1}{\Sigma_0}\bigg[ -\frac{\mathsf{k}R_y}{n_5}\,\Delta dt \wedge dy + n_5\left(n_5 \sinh^2\!\rho + n_5\,\mathsf{m}^2 + \mathsf{k}^2 R_y^2\right) \cos^2\theta\; d\phi \wedge d\psi \\
&\quad + \left(\mathsf{m}\Delta dt - \left(\mathsf{m}\frac{\mathsf{p}}{R_y} + \mathsf{n}\,\mathsf{k}R_y\right) dy\right) \wedge \cos^2\theta\, d\psi - \left(\mathsf{n}\Delta dt - \left(\mathsf{n}\frac{\mathsf{p}}{R_y} + \mathsf{m}\,\mathsf{k}R_y\right) dy\right) \wedge \sin^2\theta\, d\phi \bigg], \\
e^{2\Phi} \;&= \frac{\Delta}{\Sigma_0}\,\frac{\mathsf{k}R_y}{Q_1} \;=\; \frac{\Delta}{\Sigma_0}\,\frac{\mathsf{p}/R_y}{Q_p}\,.
\end{aligned}
$$

$$
\begin{aligned}
\Sigma_0 &= \sinh^2\rho + (\mathsf{m}^2 - \mathsf{n}^2)\cos^2\theta + \mathsf{n}^2 + \frac{\mathsf{k}^2 R_y^2}{n_5}\,, \\
\Delta &= \sqrt{n_5(\mathsf{m}^2 + \mathsf{n}^2 - 1) + \mathsf{k}^2 R_y^2 + \frac{\mathsf{p}^2}{R_y^2}}\,,
\end{aligned}
$$

In the $AdS_3$ limit, the backgrounds simplify considerably,

$$ds^2 = n_5 \Bigg[ -\frac{1}{\mathsf{k}^2} \cosh^2\!\rho \, d\tilde{t}^2 + \frac{1}{\mathsf{k}^2} \sinh^2\!\rho \, d\tilde{y}^2 + d\rho^2 + d\theta^2$$

$$+ \sin^2\!\theta \left( d\phi - \frac{\mathsf{n}}{\mathsf{k}} d\tilde{t} + \frac{\mathsf{m}}{\mathsf{k}} d\tilde{y} \right)^2 + \cos^2\!\theta \left( d\psi + \frac{\mathsf{m}}{\mathsf{k}} d\tilde{t} - \frac{\mathsf{n}}{\mathsf{k}} d\tilde{y} \right)^2 \Bigg],$$

$$B = n_5 \Bigg[ \frac{\sinh^2\!\rho + (\mathsf{m}^2 - \mathsf{n}^2) \cos^2\theta}{\mathsf{k}^2} \, d\tilde{t} \wedge d\tilde{y} + \cos^2\theta \, d\phi \wedge d\psi$$

$$+ \sin^2\theta \left( -\frac{\mathsf{n}}{\mathsf{k}} \, d\tilde{t} + \frac{\mathsf{m}}{\mathsf{k}} \, d\tilde{y} \right) \wedge d\phi + \cos^2\theta \left( \frac{\mathsf{m}}{\mathsf{k}} \, d\tilde{t} - \frac{\mathsf{n}}{\mathsf{k}} \, d\tilde{y} \right) \wedge d\psi \Bigg],$$

$$e^{2\Phi} = \frac{n_5}{Q_1} = \frac{Q_5}{Q_1} \, .$$

The holographic description of the backgrounds in the $AdS_3$ limit is a specific family of pure CFT states generated by spectral flow with parameters

$$
\begin{aligned}
\alpha &= \frac{\mathsf{m}+\mathsf{n}}{2\mathsf{k}} = \frac{2s+1}{2\mathsf{k}} = \frac{s_+}{2\mathsf{k}}\,, \\
\bar{\alpha} &= \frac{\mathsf{m}-\mathsf{n}}{2\mathsf{k}} = \frac{2\bar{s}+1}{2\mathsf{k}} = -\frac{s_-}{2\mathsf{k}}\,.
\end{aligned}
$$

The states involve  k-twisted ground states excited by filled Fermi seas.

# Heavy-light correlators from the worldsheet

Light probes interacting with heavy bound states give rise to observables containing valuable dynamical information, for instance about black hole evaporation.

Key observables in holographic models: correlation functions involving two **heavy** background states and some number of **light** operators.

For us, heavy/light means that at large central charge $c$,

$$\Delta_{\text{light}} \sim \mathcal{O}(1)\,, \qquad \Delta_{\text{heavy}} \sim c\,.$$

Denoting a heavy state by $|H\rangle$ and a set of light operators by $\mathcal{O}_i$ , heavy-light holographic CFT correlators take the form

$$\langle H|\,\mathcal{O}_1\cdots\mathcal{O}_n\,|H\rangle\,.$$

In the $AdS_3$ limit, such correlators correspond to n-point functions of light operators in our worldsheet cosets.

We analyze the string spectrum, focusing primarily on massless states, in both NS-NS and R-R sectors.

Recall that before gauging we have building blocks of SL(2,R) and SU(2),

$$\mathcal{G}/\mathcal{H} \quad = \quad \frac{\mathrm{SL}(2,\mathbb{R})_{n_5} \times \mathrm{SU}(2)_{n_5} \times \mathbb{R}_t \times \mathrm{S}^1_y}{\mathbb{R} \times \mathrm{U}(1)} \times \mathrm{T}^4$$

The gauging gives rise to additional terms in the BRST procedure, which project out some states of the ungauged model.

The BRST charge contains the fermionic superpartner of the gauge current,

$$\boldsymbol{\lambda} = \psi^3 + l_2\chi^3 + l_3\lambda^t + l_4\lambda^y \qquad \leftrightarrow \qquad \mathcal{J} = J_3^{\mathrm{sl}} + l_2 J_3^{\mathrm{su}} + l_3\partial t + l_4\partial y$$

and takes the form

$$\mathcal{Q} = \oint dz : \left[c\left(T + T_{\beta\gamma\bar{\beta}\bar{\gamma}}\right) + \gamma G + \tilde{c}J + \tilde{\gamma}\boldsymbol{\lambda} + \mathrm{ghosts}\right] : ,$$

where the tilded ghosts are those of the null gauging procedure.

Martinec, Massai, DT '20

Vertex operators are built on the center-of-mass wavefunctions
(we will set the y-winding to zero for this talk)

$$\mathcal{V}_{\rm com} = V_{jm\bar{m}} V'_{j'm'\bar{m}'} e^{-iEt+iP_y y}$$

SL(2,R) quantum numbers

SU(2) quantum numbers

In SL(2,R) we will focus on principal discrete series representations, and for this talk we will not consider worldsheet spectral flow.

$$\mathcal{D}_j^{\pm} = \{|j,m\rangle \ , \ m = \pm j, \pm j \pm 1, \pm j \pm 2, \cdots \} \, ,$$

Then, for instance, NS-NS supergravity states must satisfy the Virasoro constraint

$$0 = -\frac{j(j-1)}{n_5} + \frac{j'(j'+1)}{n_5} - \frac{1}{4}E^2 + \frac{1}{4}P_y^2$$

and bosonic null constraints

$$0 = m + l_2\, m' + \frac{l_3}{2}E + \frac{l_4}{2}P_y\,, \qquad\qquad 0 = \bar{m} + r_2\, \bar{m}' + \frac{r_3}{2}E + \frac{r_4}{2}P_y\ .$$

NS-NS sector: before BRST, 8 polarizations (ignoring $T^4$);

2 combinations removed by the G and $\lambda$ constraints; 2 are BRST exact.

$\rightarrow$ 4 physical polarizations for generic $j$, as is correct for (5+1)-d physical spacetime. (8 if we re-include the $T^4$).

NS-NS sector: before BRST, 8 polarizations (ignoring $T^4$);

2 combinations removed by the G and $\lambda$ constraints; 2 are BRST exact.

→ 4 physical polarizations for generic $j$, as is correct for (5+1)-d physical spacetime. (8 if we re-include the $T^4$). For $\varepsilon = \pm 1$ these 4 are given by

$$
\begin{aligned}
\mathcal{W}^{\varepsilon} &= e^{-\varphi}\left[(\psi V_j)_{j+\varepsilon,m} V'_{j'm'} + \left(c^t_\varepsilon\,\lambda^t + c^y_\varepsilon\,\lambda^y\right) V_{jm} V'_{j'm'}\right] e^{i(-E\,t+P_y y)}\,,\\
\mathcal{X}^{\varepsilon} &= e^{-\varphi}\left[V_{jm} (\chi V'_{j'})_{j'+\varepsilon,m'} + \left(d^t_\varepsilon\,\lambda^t + d^y_\varepsilon\,\lambda^y\right) V_{jm} V'_{j'm'}\right] e^{i(-E\,t+P_y y)}\,.
\end{aligned}
$$

$$
c^t_\varepsilon = -c^3_\varepsilon \frac{n_5 P_y}{l_4 E + l_3 P_y} \qquad\qquad c^y_\varepsilon = c^3_\varepsilon \frac{n_5 E}{l_4 E + l_3 P_y}\,,
$$

$$
d^t_\varepsilon = d^3_\varepsilon \frac{n_5 l_2 P_y}{l_4 E + l_3 P_y} \qquad\qquad d^y_\varepsilon = -d^3_\varepsilon \frac{n_5 l_2 E}{l_4 E + l_3 P_y}\,.
$$

Bufalini, Iguri, Kovensky, DT ’22

R-R sector:  start with 64-component spinor of O(10,2);

Analog of GSO in (10+2) dimensions cuts this down to 32;

BRST cuts this down by a further factor of 4, leaving 8 physical polarizations.

We use (-1/2,+1/2) picture for ($\varphi$, $\tilde{\varphi}$ ) ghosts.

( $\tilde{\varphi}$  arises from bosonization of  $\tilde{\beta}$ , $\tilde{\gamma}$  ghosts for null gauging).

R-R sector: start with 64-component spinor of O(10,2);

Analog of GSO in (10+2) dimensions cuts this down to 32;

BRST cuts this down by a further factor of 4, leaving 8 physical polarizations.

We use (-1/2,+1/2) picture for ($\varphi$, $\tilde{\varphi}$ ) ghosts.

( $\tilde{\varphi}$ arises from bosonization of $\tilde{\beta}$ , $\tilde{\gamma}$ ghosts for null gauging).

We start with an ansatz of the form

$$\mathcal{Y}^{\varepsilon_4,\varepsilon_5} = e^{-(\varphi-\tilde{\varphi})/2} \sum_{\varepsilon_1,\varepsilon_2,\varepsilon} F^{\varepsilon}_{\varepsilon_1\varepsilon_2\varepsilon_4\varepsilon_5} S_{\varepsilon_1\varepsilon_2\varepsilon_3} \mathcal{S}_{\varepsilon_6\varepsilon_4\varepsilon_5} V_{j,m-\frac{\varepsilon_1}{2}} V'_{j',m'-\frac{\varepsilon_2}{2}} e^{i(-Et+P_y y)} ,$$

and constrain the coefficients $F$ .

SL(2,R) x SU(2)
spin field

t, y, T[4]
spin field

So far we discussed vertex operators in the basis in which the Cartan currents are diagonalized, often known as the “m-basis”.

To study $AdS_3$ holography, one introduces a conjugate “x-basis” in which the complex label x corresponds to the local coordinate of the holographic CFT.

Maldacena, Ooguri ’00

In the $AdS_3$ limit of our models, the identification of the x coordinate requires some care, due to the gauging procedure.

As a first step, we make a systematic large $R_y$ expansion.

We choose a gauge in which the upstairs SL(2,R) time and angular direction are fixed. Then, importantly, $t/R_y$ and $y/R_y$ parametrize the asymptotic boundary of the physical $AdS_3$ of the gauged model. We define

$$m_y = \frac{1}{2}\left(\mathcal{E} + n_y\right) \qquad \bar{m}_y = \frac{1}{2}\left(\mathcal{E} - n_y\right) \ ,$$

and interpret these as the asymptotic mode labels.

We thus define the x-basis operators

$$\tilde{\mathcal{O}}_h^{(m')}(x,\bar{x}) \ \equiv \ \frac{1}{\mathsf{k}^{h+\bar{h}}} \sum_{m_y,\bar{m}_y} x^{m_y - h} \bar{x}^{\bar{m}_y - \bar{h}} \mathcal{V}_{j,m,\bar{m}} \mathcal{V}'_{j',m',\bar{m}'} e^{-im_y(\tilde{t}-\tilde{y})} e^{-i\bar{m}_y(\tilde{t}+\tilde{y})} \, .$$

$$\tilde{t} = \frac{t}{R_y} \, , \quad \tilde{y} = \frac{y}{R_y}$$

This leads to a considerable set of new results, a subset of which match very non-trivially to two families of known HLLH correlators.

Importantly, in the AdS limit, the bosonic null constraint restricts the quantum numbers as

$$0 \;=\; m + (2s+1)m' - \mathsf{k}\, m_y \;=\; \bar{m} + (2\bar{s}+1)\bar{m}' - \mathsf{k}\, \bar{m}_y \,.$$

Therefore, $m_y$ and $\bar{m}_y$ take fractional values, in multiples of 1/k.
However, momentum quantization along y imposes that $m_y - \bar{m}_y$ is an integer.
This implies that local operators are built only out of the subset of SL(2,R) and SU(2) modes that satisfy a modulo-k condition,

$$\bar{m} - m \;\equiv\; s_+ m' - s_- \bar{m}' \pmod{\mathsf{k}} \,.$$

# Example HLLH correlator

Let's take the light operator to be a particular RR supergravity state, dual to the dimension (1/2,1/2) chiral primary of the holographic CFT mentioned earlier.

We compute its two-point function in the cosets. All the non-trivial physics comes from the gauge constraints. Schematically we have

$$\langle\tilde{\mathcal{O}}_1(x_1,z_1)\tilde{\mathcal{O}}_2(x_2,z_2)\rangle \equiv \frac{1}{\mathsf{k}^{4h}} \sum_{m_{y,i},\bar{m}_{y,i}} \prod_{i=1,2} x_i^{m_{y,i}-h_i}\bar{x}_i^{\bar{m}_{y,i}-h_i} \lim_{R_y\to\infty} \langle\mathcal{V}_1(z_1)\mathcal{V}_2(z_2)\rangle_{\text{coset}}\,,$$

For this light operator, the last term is particularly simple.

We consider a discrete series rep $D_j^+$ . We have $j = h, \quad m' = \bar{m}' = -h,$
$m = h + n, \quad \bar{m} = h + \bar{n}$ , where $n, \ \bar{n}$ are non-negative integers.
Setting $h = 1/2,$ the bosonic null constraint in the AdS limit reduces to

$$m_y = \frac{n - s}{\mathsf{k}}\,, \qquad \bar{m}_y = \frac{\bar{n}}{\mathsf{k}}\,.$$

Suppressing the $z$ dependence, and writing $s = \mathsf{k}\mathsf{p} - a$ with non-negative integer p and $0 \leq a < \mathsf{k}$, the correlator takes the form

$$\langle \mathcal{Y}_{A,1}(1) \mathcal{Y}_{A,2}(x) \rangle = \frac{1}{\mathsf{k}^2} \sum_{n,\bar{n}}{}' x^{\frac{n-s}{\mathsf{k}} - \frac{1}{2}}\, \bar{x}^{\frac{\bar{n}}{\mathsf{k}} - \frac{1}{2}}$$

sum over $\bar{n} - n \equiv a \mod \mathsf{k}\,.$

This correlator agrees beautifully with the supergravity and orbifold CFT expressions in previous literature, given as ($\bar{s}$ =0 , and $s \equiv \hat{s}$ mod k)

$$\langle s,k|O_L(1)O_L^\dagger(x)|s,k\rangle \;=\; \frac{x^{(\hat{s}-s)/\mathsf{k}}}{|x||1-x|^2}\,\frac{1-|x|^{2(1-\hat{s}/\mathsf{k})}+\bar{x}\left(|x|^{-2\hat{s}/\mathsf{k}}-1\right)}{1-|x|^{2/\mathsf{k}}}\,.$$

By manipulating this expression and expanding denominators, we show that it is identical to the worldsheet expression.

Previously, agreement had been observed between supergravity and orbifold CFT, in special cases $\hat{s}=0$ , $\hat{s}=\mathsf{k}-1$ , however now we have the result exactly in $\alpha'$ and for all $\hat{s}$ .

Bufalini, Iguri, Kovensky, DT ’22

Moreover, we have generalized this HLLH correlator to the full set of heavy backgrounds, and where the light operators are any massless states.

By enforcing the mod k constraint using a Kronecker comb,

$$\sum_{q\in\mathbb{Z}} \delta_{m-\bar{m},\mathsf{k}q} = \mathsf{k}^{-1} \sum_{r=0}^{\mathsf{k}-1} e^{2\pi i r \,\frac{m-\bar{m}}{\mathsf{k}}} \,,$$

the sum over **restricted** powers of x becomes an **unrestricted** sum of powers of the $\mathsf{k}^{\text{th}}$ roots of x, yielding

$$\langle O^{++}(1) O^{--}(x)\rangle_H = \frac{1}{\mathsf{k}^3} \sum_{u^{\mathsf{k}}=x} \frac{u^{-\frac{s_+}{2}}\,\bar{u}^{-\frac{s_-}{2}}\,|u|^{1-\mathsf{k}}}{|1-u|^2} \,.$$

$$\langle O^{++}(1)O^{--}(x)\rangle_H = \frac{1}{\mathsf{k}^3}\sum_{u^{\mathsf{k}}=x}\frac{u^{-\frac{s_+}{2}}\,\bar{u}^{-\frac{s_-}{2}}\,|u|^{1-\mathsf{k}}}{|1-u|^2}\,.$$

This formula is readily generalized to generic massless insertions;

Moreover, the appearance of the $\mathrm{k}^{\mathrm{th}}$ roots of $x$ invites a comparison with the holographically dual symmetric product orbifold CFT.

Let us consider a worldsheet operator with definite charge $m'$, and write

$$m_y = x\partial_x + h\,, \qquad m = u\partial_u + h - \beta$$

for auxiliary $\beta$ . Then to obtain $u^k = x$ we require

$$\beta = h(1-\mathsf{k}) + s_+ m'\,, \quad \bar{\beta} = h(1-\mathsf{k}) + s_- \bar{m}'\,.$$

Then the x-basis worldsheet operator becomes

$$O_h(x) = \frac{1}{\mathsf{k}^{2h+1}} \sum_{u^{\mathsf{k}}=x} u^{\beta}\bar{u}^{\bar{\beta}} \mathcal{V}_h(u) \mathcal{V}'_{h'm'\bar{m}'}\,.$$

We thus find that worldsheet HLLH correlators with generic massless insertions take the surprisingly simple form

$$\langle O_L(x_1)\bar{O}_L(x_2)\rangle_H = \frac{1}{\mathsf{k}^{4h+2}} \sum_{u_i^{\mathsf{k}}=x_i} \frac{u_1^{\beta_1}\,\bar{u}_1^{\bar{\beta}_1}\,u_2^{\beta_2}\,\bar{u}_2^{\bar{\beta}_2}}{|u_1-u_2|^{4h}}\,.$$

A priori, this gives a prediction for the holographic CFT at strong coupling.

However, we have already seen one example where the correlator is protected between supergravity, worldsheet, and holographic CFT.

Remarkably, in all other cases that we have been able to check, we also find agreement with the holographic CFT.

Furthermore, we conjecture a formula for worldsheet correlators with n massless insertions on these backgrounds:

$$\langle O_1(x_1)\dots O_n(x_n)\rangle_H \;=\; \frac{1}{\mathsf{k}^{2\mathsf{H}+n}}\sum_{u_i^{\mathsf{k}}=x_i}\left(\prod_{i=1}^{n} u_i^{\beta_i}\,\bar{u}_i^{\bar{\beta}_i}\right)\langle \hat{O}_1(u_1)\dots \hat{O}_n(u_n)\rangle\,.$$

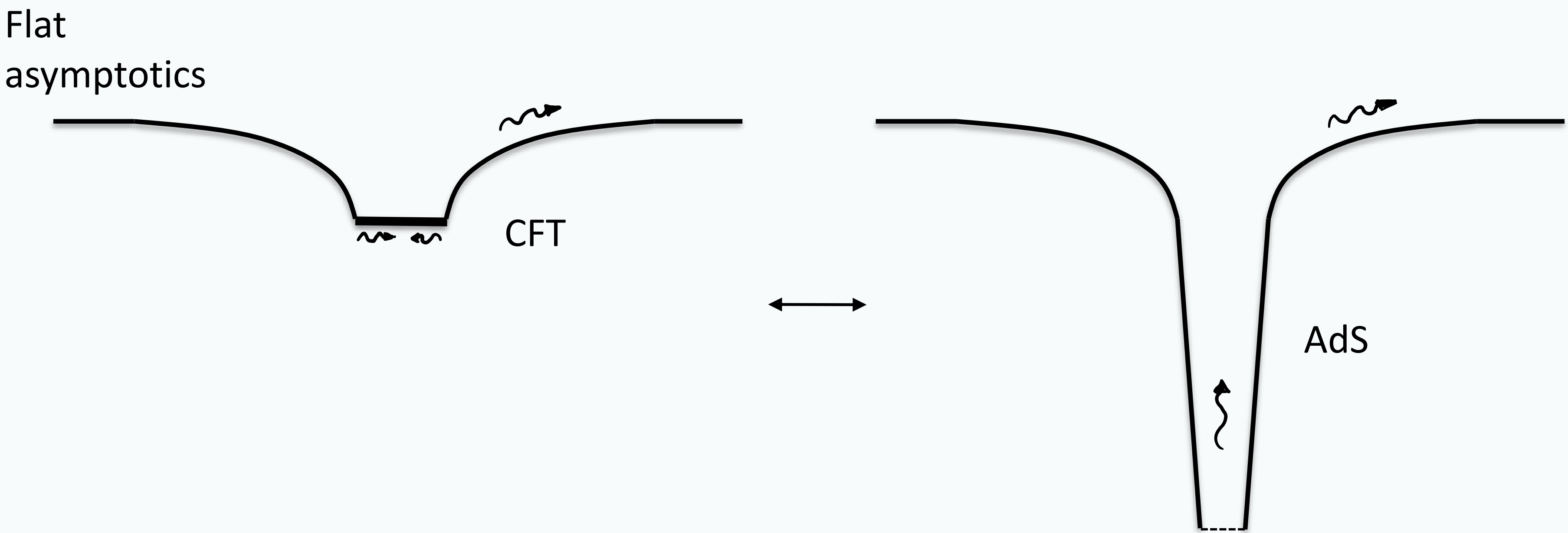
This is a worldsheet expression;
We do not expect agreement with the holographic CFT beyond 3 light insertions.

However we have checked a HLLLH five-point function, and we again find precise agreement with the holographic CFT.

This remarkable agreement is presumably due to the special nature of the heavy states under consideration.

Bufalini, Iguri, Kovensky, DT ’22

# Unitary analog of Hawking radiation



By taking a limit of the two-point function in the non-BPS backgrounds, we obtain the amplitude for the unitary analog of Hawking radiation from these backgrounds,

$$\mathcal{A}(x) = \frac{1}{\mathsf{k}^{2h}} \frac{\sum_{\ell\in\mathbb{Z}} \delta_{s_+ m' - s_- \bar{m}', \mathsf{k}\ell}}{x^{h(1+\frac{1}{\mathsf{k}}) - m' \frac{s_+}{\mathsf{k}}} \; \bar{x}^{h(1+\frac{1}{\mathsf{k}}) - \bar{m}' \frac{s_-}{\mathsf{k}}}} \ ,$$

again in precise agreement with supergravity and holographic CFT.

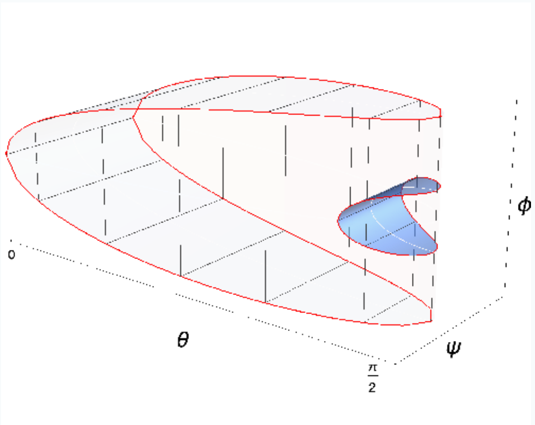
# D-brane probes and general gauged models

# D-brane probes of black hole microstates

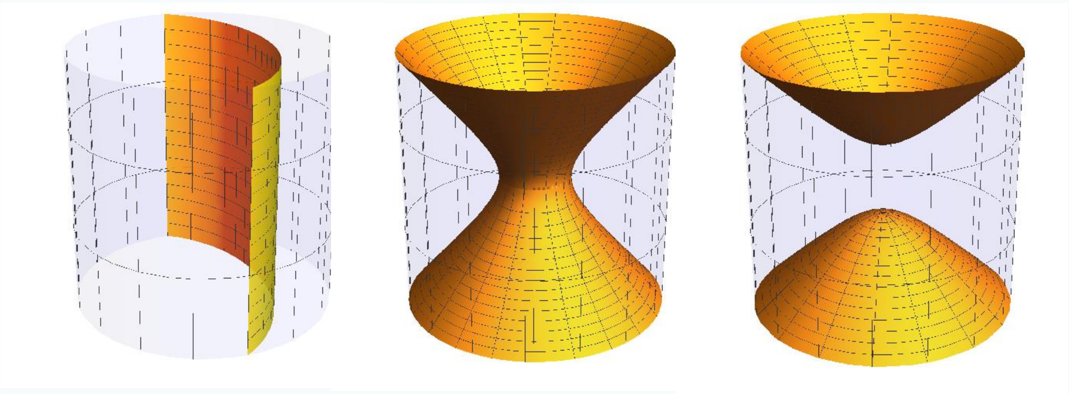
D-brane spectrum: determined by consistent boundary conditions on worldsheet

For WZW model on a group $\mathcal{G}$, a simple example is a boundary that is a (twisted) conjugacy class of $\mathcal{G}$, specified by a fixed element $f_{\mathcal{G}}$ and an automorphism $\Omega_{\mathcal{G}}$ – known as "symmetry-preserving branes".

$$\mathcal{C}_{\mathcal{G}}^{(f_{\mathcal{G}},\Omega_{\mathcal{G}})} \equiv \{g f_{\mathcal{G}}\, \Omega_{\mathcal{G}}(g^{-1})\ ,\quad g \in \mathcal{G}\}$$

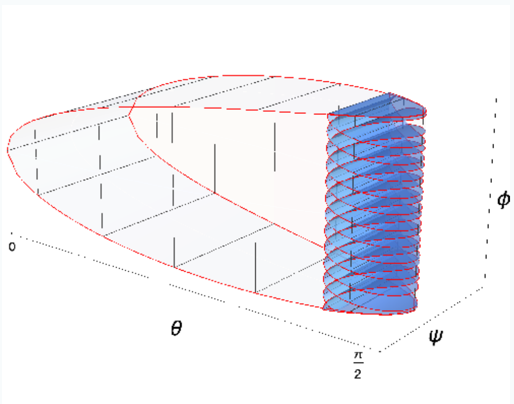


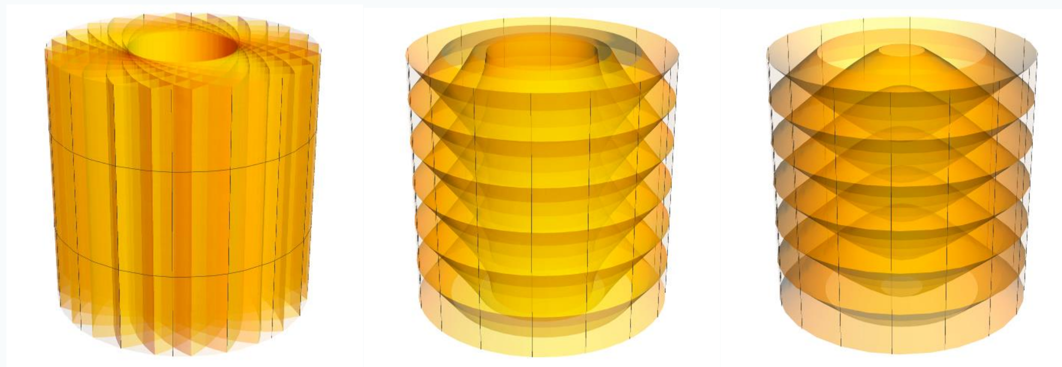

$S^2 \in SU(2)$



$\mathrm{AdS}_2\ ,\quad \mathrm{dS}_2\ ,\quad \mathbb{H}_2 \quad \in\ SL(2,\mathbb{R})$

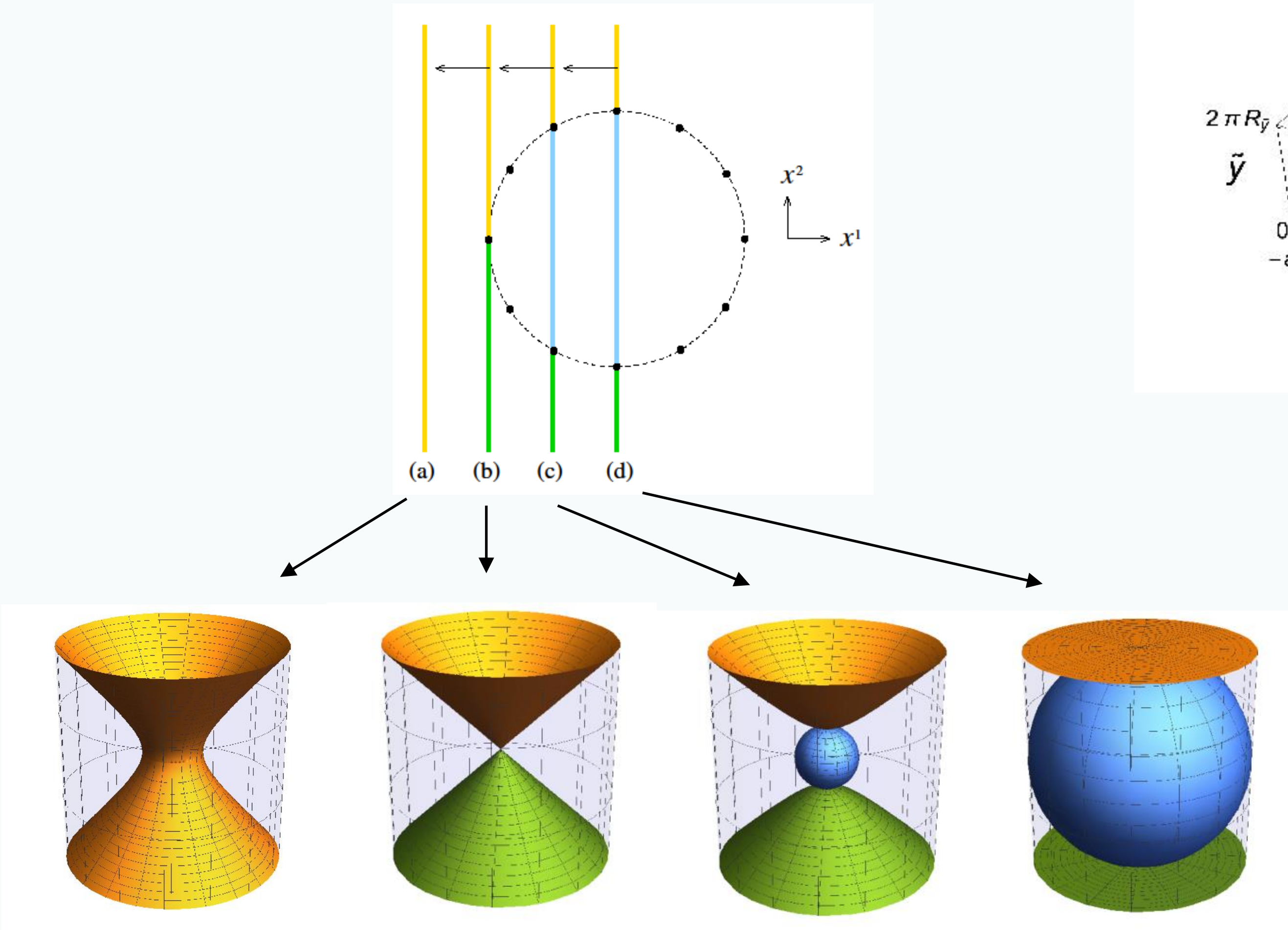
More generally, given a subgroup $\mathcal{H}$ of $\mathcal{G}$, there are brane worldvolumes that are a product of a conjugacy class of $\mathcal{G}$ and a conjugacy class of $\mathcal{H}$, embedded in $\mathcal{G}$. This has the effect of smearing the above branes, and preserving only $\mathcal{H}$ on the boundary: "symmetry-breaking" branes.

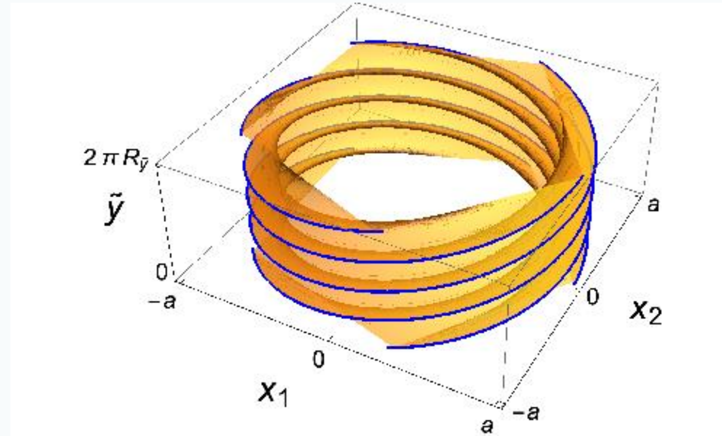


In a gauged sigma model, the branes that survive the gauging are those that, before gauging, are extended in the gauge directions. We smear as necessary.

We find a rich spectrum of branes that probe the sub-string-scale structure of the background. Examples:




Martinec, Massai, DT '19

# More general gauged models

We have shown that the null-gauging formalism can be extended

to the larger family of two-charge Lunin-Mathur black hole microstates.

Generically, these gauged models are not cosets.

However much can still be analyzed explicitly.

For instance, one can construct elliptically deformed backgrounds.

Martinec, Massai, DT '20

# Summary

We have constructed and studied families of worldsheet models that describe black hole microstates.

These models capture physics beyond supergravity such as the fine microstructure of the bound state and the physics of probe strings and D-branes.

We recently computed several heavy-light correlators using these models, finding better-than-expected agreement with the holographic CFT.

These developments offer the tantalizing prospect of understanding the most entropic part of the black hole Hilbert space.

Thanks!