OBSERVABLES OF O-C (SUPER)-SFT

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(to appear)

SFT @ Prague 2022

§0. MOTIVATION

Want to construct observables which would help with identifying classical solutions in O-C (super) SFT

$\sim$ in RNS formalism

$$\left(\mathrm{CFT},\, g_s,\, \|B\rangle\!\rangle\right) \xrightarrow[\Phi^*,\ \Psi^*]{} \left(\widehat{\mathrm{CFT}},\, \hat{g}_s,\, \|\hat{B}\rangle\!\rangle\right)$$

soln. in (pure-sphere) L_∞ SFT

soln in (pure-disk) weak-A_∞ SFT

off-shell superstring disk amps with ≥ 1 open string sufficient

[Kunitomo '22]

Feasible solutions $\Phi^* \neq 0$, Ψ^* (at the moment): marginal defs.

- Narain moduli space $\quad \varepsilon_{ij}\, c\bar{c}\, \partial X^i \bar{\partial} X^j$
- ghost dilaton $\quad D\left(c\partial^2 c - \bar{c}\bar{\partial}^2\bar{c}\right)$
- orbifold blow-ups

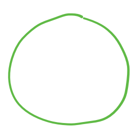
$\vdots$

Gauge-invariant quantities provided by:

on-shell amps for fluctuations ϕ, ψ around Φ^*, Ψ^*

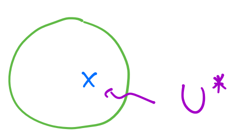
Simplest non-trivial amps:

- 0-pt disk $(= Z_D)$



$\Rightarrow$ O-C disk action evaluated on Φ^*, Ψ^* [see Carlo's talk]

- 1-pt disk



$\Rightarrow$ generalization of Ellwood invariant in OSFT

[Hashimoto, Itzhaki '01; Ellwood '09]

↳ need off-shell superstring disk amps with $\mathcal{O}$ open strings

'Munich-like' construction in SHS ?

[Erler, Konopka, Sachs '13]

§1. CLASSICAL O-C OBSERVABLES FROM BACKGROUND INDEPENDENCE

Tree-level truncation of the full quantum O-C SFT action:

$$S(\Phi;\Psi) =$$

$$= \frac{1}{g_s^2} \sum_{j=0}^{\infty} \frac{1}{(j+1)!}\, \omega_c\big(\Phi, l_j(\Phi^{\wedge j})\big) \quad \Big\} \text{ spheres} \equiv S_S(\Phi)$$

$$+ \frac{1}{g_s}\Bigg(\sum_{j=1}^{\infty} \frac{1}{(j+1)!}\, \omega_c\big(\Phi, l_{j,0}(\Phi^{\wedge j})\big) +$$

only bulk punctures

$$+ \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} \frac{1}{j!} \frac{1}{k+1}\, \omega_o\big(\Psi, m_{j,k}(\Phi^{\wedge j}; \Psi^{\otimes k})\big)\Bigg) \quad \Big\} \text{ disks} \equiv S_D(\Phi;\Psi)$$

at least 1 bdy puncture

$$+ \mathcal{O}(g_s^0) \quad \longleftarrow \text{ quantum corrections}$$

Consistent classical background:

$$\big(\; \mathrm{CFT} \; , \; \| B \rangle\rangle \; , \; g_s \; \big)$$

bulk matter + ghost $c_{tot} = 0$

conformal boundary state ($\sim l_{0,0}$)

closed-string coupling const.

$$\Rightarrow \quad \begin{aligned} l_0 &= 0 \\ m_{0,0} &= 0 \end{aligned}$$

$l_0 = 0$ ← closed-string sphere tadpole

$m_{0,0} = 0$ ← open-string disk tadpole

Equivalently, can recast the disk-part of the action as

$$S_D(\Phi;\Psi) = \frac{1}{g_s}\Bigg(\sum_{k=0}^{\infty} \frac{1}{k+1}\, \omega_o\big(\Psi, m_{0,k}(\Psi^{\otimes k})\big) \; +$$

only bdy punctures

$$+ \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} \frac{1}{(j+1)!}\frac{1}{k}\, \omega_c\big(\Phi, l_{j,k}(\Phi^{\wedge j}; \Psi^{\otimes k})\big) \Bigg)$$

at least 1 bulk puncture

where we define $l_{j,k}$ s.t.

$$\omega_c\big(\Phi_0, l_{j,k}(\Phi_1 \wedge .. \wedge \Phi_j ; \Psi_1 \otimes .. \otimes \Psi_k)\big) =$$

$$= -(-1)^{d(\Phi_0)}\, \omega_o\big(m_{j+1,k-1}(\Phi_0 \wedge .. \wedge \Phi_j ; \Psi_1 \otimes .. \otimes \Psi_k), \Psi_{k+1}\big)$$

Impose quantum BV master equation and solve order-by-order in g_s :

$$\text{SDHA}\left\{\begin{aligned}
&\left.\begin{aligned}
0 &= \sum_{k=1}^{r-1} \ell_k\, \ell_{r-k} \qquad \rightsquigarrow L_\infty\text{-algebra} \\
0 &= \sum_{k=1}^{r} m_{k,s-1}\, \ell_{r+1-k} + \sum_{k\geq 0}^{r} \sum_{n=1}^{s} m_{k,n}\, m_{r-k,s-n}
\end{aligned}\right\}\ \text{OCHA} \\
&\ 0 = \sum_{k=1}^{r} [\ell_k, \ell_{r-k,0}] + \sum_{k=1}^{r-1} \ell_{k-1,1}\, m_{r-k,0}
\end{aligned}\right.$$

$$\vdots$$

or, equivalently

$$0 = \sum_{k=1}^{r-1} \ell_k\, \ell_{r-k} \qquad \rightsquigarrow L_\infty\text{-algebra}$$

$$0 = \sum_{n=1}^{s-1} m_{0,n}\, m_{0,s-n} \qquad \rightsquigarrow A_\infty\text{-algebra}$$

$$0 = \sum_{k=1}^{r} [\ell_k, \ell_{r-k,s}] + \sum_{k=1}^{r} \sum_{n=0}^{s} \ell_{k-1,n+1}\, m_{r-k,s-n}$$

$$\vdots$$

Ex. 1 : examples of SDHA relations

- $m_{1,0} \underbrace{\ell_1}_{Q_c} + \underbrace{m_{0,1}}_{Q_o} m_{1,0} = 0$

- $\underbrace{\ell_1}_{Q_c} \ell_{0,1} + \ell_{0,1} \underbrace{m_{0,1}}_{Q_o} = 0$

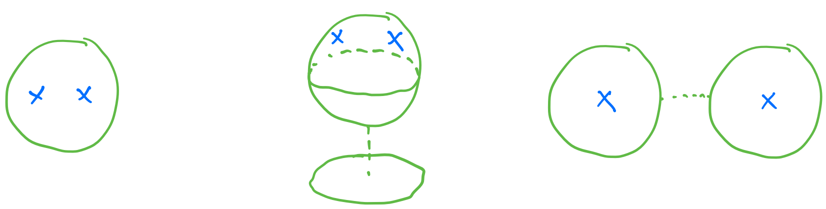
} BRST charge passes through the elementary O-C product

Non - OCHA :

- $\ell_1 \ell_{0,0} = 0$

  ~ boundary state (up to a stub) is BRST closed

- $[\ell_1, \ell_{1,0}] + \ell_2 \ell_{0,0} + \ell_{0,1} m_{1,0} = 0$



$\square$

Classical equations of motion:

1. vary the sphere action wrt Φ: L_∞ structure

$$0 = \sum_{j=1}^{\infty} \frac{1}{j!} \, l_j(\Phi^*)$$

$\rightsquigarrow$ Φ^* interpolates $(\mathrm{CFT}, g_s) \to (\widehat{\mathrm{CFT}}, \hat{g}_s)$

↳ Φ^* determined up to the L_∞ gauge variation

$$\delta_\Omega \Phi^* = \sum_{j=0}^{\infty} \frac{1}{j!} \, l_{1+j}(\Omega \wedge \Phi^{*\wedge j})$$

$\in \mathcal{H}_c$, degree-odd gauge parameter

2. substitute $\Phi = \Phi^*$ in the disk action $\rightsquigarrow$ weak-A_∞ structure

and vary wrt Ψ:

$$0 = \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} \frac{1}{j!} \, m_{j,k}(\Phi^{*\wedge j} ; \Psi^{*\otimes k})$$

$\rightsquigarrow$ Ψ^* interpolates $\mathrm{BCFT} \to \widehat{\mathrm{BCFT}}$

$\Rightarrow$ Ψ^* gives the open-string vacuum shift

↳ Ψ^* determined up to the gauge variations

- $$\delta_\Lambda \Psi^* = \sum_{j,k,r} \frac{1}{j!}\, m_{j,k+1}\left(\Phi^{*\wedge j};\ \Psi^{*\otimes r} \otimes \Lambda \otimes \Psi^{*\otimes k-r}\right)$$

  $\in \mathcal{H}_o$, degree-odd gauge parameter

- $$\delta_\Omega \Psi^* = \sum_{j,k} \frac{1}{j!}\, m_{j+1,k}\left(\Omega \wedge \Phi^{*\wedge j};\ \Psi^{*\otimes k}\right)$$

Schematically therefore have:

$$\left(\mathrm{CFT},\ \|B\rangle\rangle,\ g_s\right) \xrightarrow{\ \Phi^*,\ \Psi^*\ } \left(\widehat{\mathrm{CFT}},\ \|\hat{B}\rangle\rangle,\ \hat{g}_s\right)$$

Φ^*, Ψ^* determined only by OCHA

↳ need some observables to identify $\left(\widehat{\mathrm{CFT}},\ \|\hat{B}\rangle\rangle,\ \hat{g}_s\right)$

functions of Φ^*, Ψ^* invariant under $\delta_\Omega, \delta_\Lambda$

$\Rightarrow$ provided by on-shell scattering amps of fluctuations around Φ^*, Ψ^*

$$\Updownarrow \text{ B.I.}$$

on-shell scattering amps for $\left(\widehat{CFT}, |\hat{B}\rangle\rangle, \hat{g}_s\right)$

Expand the action around Φ^*, Ψ^* :

$$\Phi = \Phi^* + \phi \qquad \Psi = \Psi^* + \psi$$

fluctuations around the
new background $(CFT^*, |B^*\rangle\rangle, g_s^*)$

$$S(\Phi^* + \phi \,;\, \Psi^* + \psi) \equiv S^*(\phi\,;\,\psi) =$$

$$= \underbrace{S_S(\Phi^*)}_{\text{on-shell sphere action}} + \frac{1}{g_s^2} \sum_{j=0}^{\infty} \frac{1}{(j+1)!}\, \omega_c\left(\phi\,,\, l_j^*(\phi^{\wedge j})\right)$$

$$+ \underbrace{S_D(\Phi^*; \Psi^*)}_{\text{on-shell disk action}} + \frac{1}{g_s}\Big(\sum_{j=1}^{\infty} \frac{1}{(j+1)!} \omega_c(\phi, l^*_{j,0}(\phi^{\wedge j})) +$$

$$+ \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} \frac{1}{j!} \frac{1}{k+1} \omega_o(\psi, m^*_{j,k}(\phi^{\wedge j}; \psi^{\otimes k})) \Big)$$

$$+ O(g_s^0)$$

↳ shifted products:

$$l_k^*(\phi_1 \wedge \ldots \wedge \phi_k) = \sum_{j=0}^{\infty} \frac{1}{j!} l_{j+k}(\Phi^{*\wedge j} \wedge \phi_1 \wedge \ldots \wedge \phi_k)$$

$$l_{k,0}^*(\phi_1 \wedge \ldots \wedge \phi_k) = \sum_{j=0}^{\infty} \frac{1}{j!} \frac{1}{r} l_{j+k,r}(\Phi^{*\wedge j} \wedge \phi_1 \wedge \ldots \wedge \phi_k; \Psi^{*\otimes r})$$

$$m_{k,l}^*(\phi_1 \wedge \ldots \wedge \phi_k; \psi_1 \otimes \ldots \otimes \psi_l) =$$

$$= \sum_{j=0}^{\infty} \frac{1}{j!} \sum_{\substack{r_1, r_2, \ldots \\ r_1 + r_2 + \ldots = r}} m_{j+k, r+l}(\Phi^{*\wedge j} \wedge \phi_1 \wedge \ldots \wedge \phi_k; \Psi^{*\otimes r_1} \otimes \psi_1 \otimes \Psi^{*\otimes r_2} \otimes \ldots)$$

again satisfy SDHA with

$$l_0^* = \sum_{j=0}^{\infty} \frac{1}{j!} l_j(\Phi^{*\wedge j}) = 0$$

closed SFT eom

$$m_{0,0}^* = \sum_{j=0}^{\infty} \sum_{r=0}^{\infty} \frac{1}{j!} m_{j,r}(\Phi^{*\wedge j}; \Psi^{*\otimes r}) = 0$$

open SFT vacuum shift eom

Using the shifted products, can economically rewrite

$$\delta_\Omega \Phi^* = l_1^*(\Omega)$$

as well as

$$\delta_\Omega \Psi^* = m_{1,0}^*(\Omega)$$
$$\delta_\Lambda \Psi^* = m_{0,1}^*(\Lambda)$$

On-shell scattering amps for $(\widehat{CFT}, \llbracket \hat{3} \rrbracket, \hat{j}_s)$ calculated using the Feynman rules from $S^*(\phi; \psi)$.

$\hookrightarrow$ starting point: elements $U^* \in \mathcal{H}_c$, $A^* \in \mathcal{H}_o$ of the shifted cohomology

$$\left.\begin{aligned} l_1^*(U^*) &= 0 \\ m_{0,1}^*(A^*) &= 0 \end{aligned}\right\} \text{ implicit dependence on } \Phi^*, \Psi^*$$

correspond to some $\hat{U} \in \hat{\mathcal{H}}_c$ and $\hat{A} \in \hat{\mathcal{H}}_o$ in the 'new' $\widehat{CFT}$

Applying δ_Ω and δ_Λ variations, obtain

$$\delta_\Omega U^* = l_2^*(\Omega, U^*)$$

as well as

$$\delta_\Lambda A^* = m_{0,2}^*(\Lambda, A^*) + m_{0,2}^*(A^*, \Lambda)$$

$$\delta_\Omega A^* = m_{1,1}^*(\Omega; A^*)$$

Ex. 2 : 3-pt sphere amp

$$\mathcal{A}_3^* = \frac{1}{g_s^2}\frac{1}{3!}\, \omega_c\left(U^*, l_2^*(U^*, U^*)\right)$$

$$\left(\overset{\text{B.I.}}{=} \frac{1}{\hat{g}_s^2}\frac{1}{3!}\, \hat{\omega}_c\left(\hat{U}, \hat{l}_2(\hat{U}, \hat{U})\right) = \hat{\mathcal{A}}_3 \right)$$

computed in the 'new' background $(\widehat{CFT}, \hat{g}_s)$

apply δ_Ω :

$$g_s^2\, \delta_\Omega \mathcal{A}_1^* = \frac{1}{2!}\, \omega_c\Big(\underbrace{\delta_\Omega U^*}_{= l_2^*(\Omega, U^*)},\ l_2^*(U^*, U^*)\Big)$$

$$+ \frac{1}{3!}\, \omega_c\Big(U^*,\ \underbrace{(\delta_\Omega l_2^*)(U^*, U^*)}_{\substack{= l_3^*(l_1^*(\Omega),\, U^{*\wedge 2}) \\ = (-\cancel{l_1^* l_3^*} - l_2^* l_2^*)(\Omega,\, U^{*\wedge 2})}}\Big)$$

$$= \frac{1}{2!}\, \omega_c\big(\Omega,\ l_2^*(U^*,\ l_2^*(U^*, U^*))\big)$$

$$- \frac{1}{3!}\, \omega_c\big(U^*,\ l_2^* l_2^*(\Omega, U^{*\wedge 2})\big)$$

$$= \underset{\substack{\uparrow \\ \text{using cyclicity etc.}}}{\ldots} = 0 .$$

$\square$

For higher-pt amps need to know how to vary propagators.

E.g. closed-string prop: h_c^* s.t.

$$l_1^* h_c^* + h_c^* l_1^* = 1 - P_c^* \quad \longleftarrow \text{proj. on closed-string cohom.}$$

↳ use homological perturbation theory to derive $\delta_\Omega h_c^*$:

$$h_c^* + \delta_\Omega h_c^* = \frac{1}{1 + h_c^* \delta_\Omega l_1^*} h_c^*$$

$$\Rightarrow \delta_\Omega h_c^* \cdot = -[h_c^*, l_2^*(\Omega \wedge \cdot)]$$

$$- \underbrace{[l_1^*, h_c^* l_2(\Omega \wedge h_c^* \cdot)]}_{l_1^*\text{-exact} \rightsquigarrow \text{decouples}} + O(\Omega^2)$$

Ex. 2 : 4-pt sphere amp

$$\mathcal{A}_4^* = \frac{1}{g_s^2}\frac{1}{4!}\,\omega_c\left(U^*, \tilde{\ell}_3^*(U^{*\wedge 3})\right)$$

$$= \frac{1}{g_s^2}\frac{1}{4!}\Big[\omega_c\left(U^*, \ell_3^*(U^{*\wedge 3})\right) +$$

$$- 3\,\omega_c\left(U^*, \ell_2^*\left(h_c^*\ell_2^*(U^{*\wedge 2}), U^*\right)\right)\Big]$$

$$\left(\overset{\text{B.I.}}{=} \hat{\mathcal{A}}_4 \text{ computed in } (\widehat{CFT}, \hat{g}_s)\right)$$

$\square$

In practice want observables which are easy to evaluate.

- 0-pt sphere amp:

$$\hat{Z}_S = Z_S + S_S(\Phi^*)$$

shift in the sphere partition fn between (CFT, g_s) and $(\widehat{\mathrm{CFT}}, \hat{g}_s)$

on-shell closed SFT action:

$$S_S(\Phi^*) = \frac{1}{g_s^2} \sum_{j=1}^{\infty} \frac{1}{(j+1)!} \omega_c\left(\Phi^*, \ell_j(\Phi^{*\wedge j})\right)$$

$$= 0 \quad \text{up to bdy contributions} \quad \text{[Erler '21]}$$

- 0-pt disk amp: ~ disk partition function

$$\underbrace{\hat{Z}_D}_{= -\frac{1}{2\pi^2 \hat{g}_s}\langle 0 \| \hat{B}\rangle\rangle} = \underbrace{Z_D}_{= -\frac{1}{2\pi^2 g_s}\langle 0 \| B\rangle\rangle} + S_D(\Phi^*; \Psi^*)$$

shift in the disk partition fn between (BCFT, g_s) and $(\widehat{\mathrm{BCFT}}, \hat{g}_s)$

↳ on-shell disk action / (disk) cosmological const.:

also non-OCHA products !

$$S_D(\Phi^*;\Psi^*) = \frac{1}{g_s}\Bigg(\sum_{j=0}^{\infty} \frac{1}{(j+1)!}\,\omega_c\big(\Phi^*, \ell_{j,0}(\Phi^{*\wedge j})\big)$$

$$+ \sum_{j=0}^{\infty}\sum_{k=0}^{\infty} \frac{1}{j!}\,\frac{1}{k+1}\,\omega_o\Big(\Psi^*, m_{j,k}\big(\Phi^{*\wedge j};\Psi^{*\otimes k}\big)\Big)\Bigg)$$

$$\equiv \Lambda(\Phi^*;\Psi^*)$$

can be used to calculate the g-function of $\|\hat{B}\rangle\!\rangle$

[see Carlo's talk for details]

Want to compute also other boundary state coeffs of $\|\hat{B}\rangle\!\rangle$.

- 1-pt disk amp:

$$\frac{1}{g_s}\,\omega_c\left(U^*, \ell_{0,0}^*\right) = \frac{1}{g_s}\,\omega_c\left(U^*, \ell_{0,0}\right) + \underbrace{\begin{cases} + \dfrac{1}{g_s}\displaystyle\sum_{j=1}^{\infty}\frac{1}{j!}\,\omega_c\left(U^*, \ell_{j,0}(\Phi^*)\right) + \\ + \dfrac{1}{g_s}\displaystyle\sum_{j=0}^{\infty}\sum_{k=1}^{\infty}\frac{1}{j!}\frac{1}{k}\,\omega_c\left(U^*, \ell_{j,k}\left(\Phi^{*\wedge j}, \Psi^{*\otimes k}\right)\right)\end{cases}}_{\text{shift of the 1-pt disk amp between } (\mathrm{BCFT}, g_s) \text{ and } (\widehat{\mathrm{BCFT}}, \hat{g}_s)}$$

also non-OCHA products !

shift of the 1-pt disk amp between (BCFT, g_s) and $(\widehat{\mathrm{BCFT}}, \hat{g}_s)$

$\Rightarrow$ cf. Ellwood invariant of OSFT

Note that:

$$\frac{1}{g_s}\,\omega_c\left(U^*, \ell_{0,0}^*\right) = \delta_\Phi S_D(\Phi\,;\Psi)\Big|_{\substack{\Phi=\Phi^*,\ \Psi=\Psi^* \\ \delta\Phi = U^*}}$$

Specialize: $\Phi^* = 0$, then

$$l_{0,0}^* = l_{0,0} + \sum_{k=1}^{\infty} \frac{1}{k} l_{0,k}\left(\Phi^{*\otimes k}\right)$$

$\rightsquigarrow$ KOZ-like boundary state

(defined up to $l_1^* = l_1$ exact terms)

In general $(\Phi^* \neq 0)$:

$$l_{0,0}^* \sim \| \hat{B} \rangle\rangle \text{ in terms of 'old' CFT states } + l_1^*(\ldots)$$

Open-string gauge variation:

$$\delta_\Lambda l_{0,0}^* = l_{0,1}^*\left(m_{0,1}^*(\Lambda)\right)$$

$$\underset{\text{using shifted OCHA}}{\rightsquigarrow} = l_1^*\left(-l_{0,1}^*(\Lambda)\right)$$

l_1^*-exact ✓

Closed-string gauge variation:

$$\delta_\Omega l_{0,0}^* = l_{1,0}^*\left(l_1^*(\Omega)\right) + l_{0,1}^*\, m_{1,0}^*(\Omega)$$

from varying Φ^* from varying Φ^*

$$\overset{\text{using shifted SDHA}}{=} l_2^*(\Omega, l_{0,0}^*) + l_1^*\left(-l_{1,0}^*(\Omega)\right)$$

compensates for $\delta_\Omega U^* \neq 0$!

l_1^*-exact ✓

indeed:

$$\delta_\Omega\left(\frac{1}{g_s}\omega_c(U^*, l_{0,0}^*)\right) = \frac{1}{g_s}\omega_c\big(\underbrace{\delta_\Omega U^*}_{=\,l_2^*(\Omega,\,U^*)}, l_{0,0}^*\big) + \frac{1}{g_s}\omega_c\big(U^*, \underbrace{\delta_\Omega l_{0,0}^*}_{=\,l_2^*(\Omega,\,l_{0,0}^*)}\big)$$

$$\overset{\text{cyclicity}}{=} 0$$

Ex. 3 : Dp - brane on a Narain lattice with modulus $E \equiv g + B$

g-function of $\|B\rangle\rangle$

$$\|B\rangle\rangle = \# \sum_{\substack{k_L, k_R \\ k_L + \Omega k_R = 0}} \exp\Big(\sum_{n>0} \frac{1}{n} \Omega_i{}^k g_{kj} \alpha^i_{-n} \bar{\alpha}^j_{-n} \Big) |k_L, k_R\rangle\rangle$$

gluing automorphism :

$$\Omega g \Omega^T = g$$

s.t.

$$\Big[(\alpha_i)_n + \Omega_i{}^j (\bar{\alpha}_j)_{-n} \Big] \|B\rangle\rangle = 0$$

want to deform

$$E \longrightarrow \hat{E} = E + \underbrace{\delta E}_{\equiv \varepsilon} \quad \rightsquigarrow \quad \begin{aligned} \delta g &= \tfrac{1}{2} (\varepsilon + \varepsilon^T) \\ \delta B &= \tfrac{1}{2} (\varepsilon - \varepsilon^T) \end{aligned}$$

Up to $O(\varepsilon)$:

$$\Phi^*(\varepsilon) = V^\varepsilon + O(\varepsilon^2)$$

$$\Psi^*(\varepsilon) = -\frac{b_0}{L_0}\bar{P}_0\, m_{1,0}(V^\varepsilon) + O(\varepsilon^2)$$

$V^\varepsilon = \varepsilon_{ij}\, c\bar{c}\, \partial X^i \bar{\partial} X^j$

need $P_0\, m_{1,0}(V^\varepsilon) = 0$

ie.

$$\ell^*_{0,0}(\varepsilon) = \ell_{0,0} + \ell_{1,0}(V^\varepsilon) + \ell_{0,1}\left(-\frac{b_0}{L_0}\bar{P}_0\, m_{1,0}(V^\varepsilon)\right) + O(\varepsilon^2)$$

Want to probe by a zero-momentum massless state [e.g. Sen '19]

$$U^*(\varepsilon) = U - \frac{b_0^+}{L_0^+}\bar{P}_0^+\, \ell_2(V^{(\varepsilon)}, U) + O(\varepsilon^2)$$

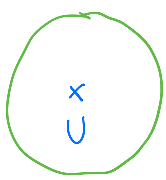
$\hat{S}$

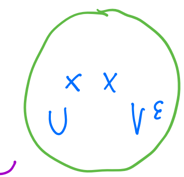
$$U = \alpha_{ij}\, c\bar{c}\, \partial X^i \bar{\partial} X^j$$

Shifted 1-pt disk amp:

$$\frac{1}{g_s}\,\omega_c\big(U^*(\varepsilon),\, l_{0,0}^*(\varepsilon)\big) =$$

$$= \frac{1}{g_s}\Big[\omega_c\big(U, l_{0,0}\big)$$





$$+ \omega_c\big(U, l_{1,0}(V^\varepsilon)\big) - \omega_c\Big(U, l_{0,1}\Big(\frac{b_0}{L_0}\bar{P}_0\, m_{1,0}(V^\varepsilon)\Big)\Big)$$

$$- \omega_c\Big(U, l_2\Big(\frac{b_0^+}{L_0^+}\bar{P}_0^+\, l_{0,0}\,,\, V^{(\varepsilon)}\Big)\Big)$$

$$+ O(\varepsilon^2)\Big]$$

Can evaluate :

$$w_c\left(U^*(\varepsilon),\, \ell^*_{0,0}(\varepsilon)\right) =$$

$$= \frac{1}{2\pi^2 g_s} \langle 0 \| B \rangle\!\rangle \Big(\frac{1}{4} \mathrm{tr}\left[\alpha \Omega^T g^{-1}\right] +$$

$$+ \frac{1}{16} \mathrm{tr}\left[\alpha \Omega^T g^{-1}\right] \mathrm{tr}\left[\varepsilon \Omega^T g^{-1}\right]$$

$$+ \frac{1}{16} \mathrm{tr}\left[\alpha g^{-1} \varepsilon^T g^{-1}\right] - \frac{1}{8} \mathrm{tr}\left[\alpha \Omega^T g^{-1} \varepsilon \Omega^T g^{-1}\right] \Big)$$

$$+ \mathcal{O}(\varepsilon^2) \qquad (\heartsuit_1)$$

At the same time

$$\frac{1}{g_s} \underbrace{w_c\left(U^*(\varepsilon),\, \ell^*_{0,0}(\varepsilon)\right)}_{\substack{\text{computed in the} \\ \text{'old' CFT Hilbert space}}} = \frac{1}{\hat{g}_s} \underbrace{\hat{w}_c\left(\hat{U},\, \hat{\ell}_{0,0}\right)}_{\substack{\text{correlator in the} \\ \text{'new' } \widehat{\text{CFT}} \text{ Hilbert space}}}$$

shifted string coupling const.

where $\hat{U}$ can receive contributions from generic massles states at zero momentum, which are excited by Φ^*:

ghost dilaton

$$\hat{U} = \underbrace{\hat{\alpha}_{ij}}_{= \alpha_{ij} + \underbrace{\delta\alpha_{ij}}_{= O(\varepsilon)}} c\bar{c}\, \partial X^i \bar{\partial} X^j + \underbrace{\hat{D}}_{\equiv \delta D = O(\varepsilon)} \left(c\partial^2 c - \bar{c}\bar{\partial}^2 \bar{c} \right)$$

ie.

$$\frac{1}{\hat{g}_s} \hat{\omega}_c(\hat{U}, \hat{\ell}_{0,0}) = \underbrace{\frac{1}{2\pi^2 \hat{g}_s} \langle 0 \| \hat{B} \rangle\!\rangle}\left(\frac{1}{4} \mathrm{tr}\left[\hat{\alpha}\, \underbrace{\hat{\Omega}^T}_{\equiv \Omega^T + \delta\Omega^T} \hat{g}^{-1} \right] + 2\hat{D} \right) \qquad (\heartsuit_2)$$

$$= \frac{1}{2\pi^2 g_s} \langle 0 \| B \rangle\!\rangle \left(1 + \frac{1}{4} \mathrm{tr}\left[\varepsilon\, \Omega^T g^{-1} \right] + O(\varepsilon^2) \right)$$

computed from $S_D(\Phi^*, \Phi^*)$ [see Carlo's talk]

Equating $(\heartsuit_1)$ with $(\heartsuit_2)$ and keeping only $O(\varepsilon)$:

$$\frac{1}{4}\mathrm{tr}\left[\alpha\,\delta\Omega^T g^{-1}\right] =$$

$$= \frac{1}{8}\mathrm{tr}\left[\varepsilon\left(1-\Omega^T\right)g^{-1}\alpha\,\Omega^T g^{-1}\right] + \frac{1}{16}\mathrm{tr}\left[\varepsilon^T g^{-1}\alpha\left(1+2\Omega^T\right)g^{-1}\right]$$

$$- \frac{1}{4}\mathrm{tr}\left[\delta\alpha\,\Omega^T g^{-1}\right] - 2\,\delta D + O(\varepsilon^2)$$

↳ can compute $\delta\Omega$ provided that we find $\delta\alpha$ and δD

- should in principle follow by comparing sphere amps
- or, can use the knowledge of $\delta\Omega$ for a (finite) number of $\|B\rangle\rangle$

$$\Rightarrow \qquad \delta\alpha = \frac{1}{2}\left(\varepsilon g^{-1}\alpha + \alpha g^{-1}\varepsilon\right)$$

$$\delta D = -\frac{1}{32}\mathrm{tr}\left[\alpha g^{-1}\varepsilon^T g^{-1}\right]$$

This gives

$$\delta\Omega = \frac{1}{2}(\varepsilon - \Omega\varepsilon^T)\, g^{-1}(1+\Omega)$$

↳ can be alternatively derived using BCFT techniques ✓

$\square$

§ 2. SUPERSTRING SDHA PRODUCTS

Want to construct the tree-level part of the O-C SFT action for the RNS superstring.

↳ need to consistently distribute PCOs

Let us focus on the NS sector.

$$\left.\begin{array}{l} \Phi \in \mathcal{H}^{S}_{c}\,, \quad \text{picture } (-1,-1) \\ \Psi \in \mathcal{H}^{S}_{o}\,, \quad \text{picture } -1 \end{array}\right\} \text{dynamical string fields}$$

Small Hilbert space:

$$\Phi \in \mathcal{H}^{S}_{c} \quad \Leftrightarrow \quad 0 = \eta_0 \Phi = \bar{\eta}_0 \Phi$$

$$\Psi \in \mathcal{H}^{S}_{o} \quad \Leftrightarrow \quad 0 = \eta_0 \Psi$$

$$S(\Phi;\Psi) =$$

constructed by [Erler, Konopka, Sachs '15]

$$= \frac{1}{g_s^2} \sum_{j=0}^{\infty} \frac{1}{(j+1)!} \, \omega_c^S\left(\Phi, L_j^{(j-1,j-1)}(\Phi^{\wedge j})\right)$$

THIS TALK

$$+ \frac{1}{g_s}\left(\sum_{j=1}^{\infty} \frac{1}{(j+1)!} \, \omega_c^S\left(\Phi, L_{j,0}^{(2j-2)}(\Phi^{\wedge j})\right) + \right.$$

$$\left. + \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} \frac{1}{j!} \frac{1}{k+1} \, \omega_0^S\left(\Psi, M_{j,k}^{(2j+k-1)}(\Phi^{\wedge j}; \Psi^{\otimes k})\right)\right)$$

constructed by [Kunitomo '22]

$$+ O(g_s^0)$$

↳ need to construct cyclic products $L_j^{(j-1,j-1)}$, $L_{j,0}^{(2j-2)}$, $M_{j,k}^{(2j+k-1)}$ acting on $\mathcal{H}_c^S$, $\mathcal{H}_0^S$ s.t.

$$0 = \sum_{k=1}^{r-1} L_k^{(k-1,k-1)} L_{r-k}^{(r-k-1,r-k-1)} \quad ✓$$

$$0 = \sum_{k=1}^{r} M_{k,s-1}^{(2k+s-2)} L_{r+1-k}^{(r-k,r-k)} + \sum_{k=0}^{r} \sum_{n=1}^{s} M_{k,n}^{(2k+n-1)} M_{r-k,s-n}^{(2r-2k+s-n-1)} \quad ✓$$

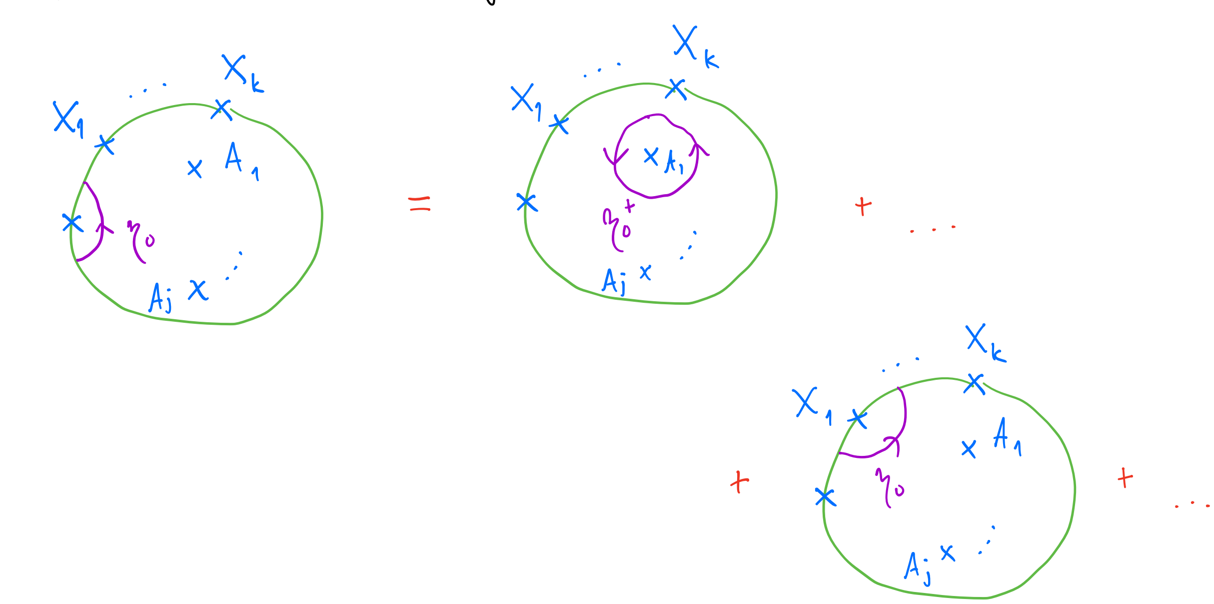
$$0 = \sum_{k=1}^{r} \left[L_k^{(k-1,k-1)}, L_{r-k,0}^{(2r-2k-2)} \right] + \sum_{k=1}^{r-1} L_{k-1,1}^{(2k-3)} M_{r-k,0}^{(2r-2k-1)} \quad ??$$

Input: degree-odd picture-0 ('bosonic') products $m_{j,k}$ s.t.

$$m_{j,k}(\underbrace{A_1,\dots,A_j}_{\in\mathcal{H}^L_c};\underbrace{X_1,\dots,X_k}_{\in\mathcal{H}^L_o}) \in \mathcal{H}^L_o \quad \rightsquigarrow \text{ contains } \xi_0,\ \bar{\xi}_0$$

is a pure-surface state with geometry giving the bosonic off-shell amp

$m_{k,l}$ does not contain any superghost contours:



$$\eta_0\, m_{j,k}(A_1,\dots,A_j;X_1,\dots,X_k) = \qquad \rightsquigarrow \eta_0^+ = \eta_0 + \bar\eta_0$$

$$= -\, m_{j,k}(\eta_0^+ A_1,\dots,A_j;X_1,\dots,X_k) - \dots$$

$$-(-1)^{d(A_1)+\dots+d(A_j)}\, m_{j,k}(A_1,\dots,A_j;\eta_0 X_1,\dots,X_k) - \dots$$

Or, concisely

$$[\eta_0, m_{j,k}] = 0$$

acts as η_0^+ on closed-string slots

$\Rightarrow$ can consistently restrict

$$m_{j,k} : \mathcal{H}_c^{-\,\wedge j} \otimes \mathcal{H}_o^{s\,\otimes k} \longrightarrow \mathcal{H}_o^{s}$$

states $A \in \mathcal{H}_c^{L}$ s.t. $\eta_0^+ A = 0$

↳ allow for $\xi_0^- \equiv \xi_0 - \bar\xi_0$

('BPZ'-dual) closed-string perspective: degree-even cyclic $\mathcal{L}^{(-1)}_{j,k}$ s.t.

$k \geq 1$!

$$\omega^L_c\left(A_0, \mathcal{L}^{(-1)}_{j,k}(A_1 \wedge \dots \wedge A_j; X_1 \otimes \dots \otimes X_k)\right) =$$

$$= -\omega^L_0\left(m_{j+1,k-1}(A_0 \wedge \dots \wedge A_j; X_1 \otimes \dots \otimes X_{k-1}), X_k\right)$$

$\Rightarrow$ $d(\omega^L_c) = d(\omega^L_0) + 1$

$\Rightarrow$ $\mathcal{L}^{(-1)}_{j,k}$ indeed degree even

Have

$$[\eta_0^r, \mathcal{L}^{(-1)}_{j,k}] = 0$$

$\Rightarrow$ can restrict

$$\mathcal{L}^{(-1)}_{j,k}: \mathcal{H}_c^{-\wedge j} \otimes \mathcal{H}_0^s \to \mathcal{H}_c^-$$

$\hookrightarrow$ 'BPZ'-dual to the restriction $m_{j,k}: \mathcal{H}_c^{-\wedge j} \otimes \mathcal{H}_0^{s\,\otimes k} \longrightarrow \mathcal{H}_0^s$

However

$$0 \neq [\eta_0^-, \mathcal{L}_{j,k}^{(-1)}] \equiv L_{j,k}^{(-2)} \equiv l_{j,k}$$ ← degree-odd

↑ acts only on closed-string slots !

$$\Rightarrow [\eta_0^\pm, l_{j,k}] = 0$$

$\Rightarrow$ can consistently restrict

$$l_{j,k} : \mathcal{H}_c^{S \wedge j} \otimes \mathcal{H}_o^{S \otimes k} \longrightarrow \mathcal{H}_c^S$$

Finally, define $\mathcal{L}_{j,0}^{(-1)}$ as pure surface states in $\mathcal{H}_c^L$ (with geometry giving an off-shell bosonic amp with only bulk insertions) and

$$[\eta_0^-, \mathcal{L}_{j,0}^{(-1)}] \equiv L_{j,0}^{(-2)} \equiv l_{j,0}$$

$\Rightarrow$ Products $m_{j,k}$, $\mathcal{L}^{(-1)}_{j,k}$ defined in terms of pure bosonic geometry:

$$0 = \sum_{k=1}^{r} m_{k,s-1}\, l_{r+1-k} + \sum_{k=0}^{r} \sum_{n=1}^{s} m_{k,n}\, m_{r-k,s-n}$$

&

$$0 = \sum_{k=1}^{r} [\, l_k , \mathcal{L}^{(-1)}_{r-k,s} \,] - \sum_{k=1}^{r} \sum_{n=0}^{s} \mathcal{L}^{(-1)}_{k-1,n+1}\, m_{r-k,s-n}$$

sic!

$\hookrightarrow$ linear in the $\mathcal{H}_c^-$ products $\mathcal{L}^{(-1)}_{j,k}$

$\Rightarrow$ can act with $[\eta_0^- , \cdot\,]$ to obtain $\mathcal{H}_c^s$ 'bosonic' homotopy relation

$$0 = \sum_{k=1}^{r} [\, l_k , l_{r-k,s} \,] + \sum_{k=1}^{r} \sum_{n=0}^{s} l_{k-1,n+1}\, m_{r-k,s-n}$$

Strategy:

1. Run a (chiral) Munich construction with ξ_0^+ starting with

$$\ell_j \quad , \qquad m_{j,k} \quad \left(\& \text{ duals } \mathcal{L}^{(-1)}_{j,k\geq 1} \right) \quad , \qquad \mathcal{L}^{(-1)}_{j,0}$$

'bosonic' products s.t. $[\eta_0^+, \cdot] = 0$

$\Rightarrow$ obtain $\mathcal{H}_c^- \otimes \mathcal{H}_0^S$ superstring products

$$L_j^{(j-1,\,j-1)} \quad , \qquad M_{j,k}^{(2j+k-1)} \quad \left(\& \text{ duals } \mathcal{L}^{(2j+k-1)}_{j,k\geq 1} \right) \quad , \qquad \mathcal{L}^{(2j-1)}_{j,0}$$

'symmetric' construction of [EKS '15]

[Kunitomo '22]

2. Apply $[\eta_0^-, \cdot]$ to obtain $\mathcal{H}_c^s \otimes \mathcal{H}_0^s$ superstring products

$$L_j^{(j-1,\, j-1)}\,, \qquad M_{j,k}^{(2j+k-1)} \quad (\text{\& duals } L_{j,k\geq 1}^{(2j+k-2)})\,, \qquad L_{j,0}^{(2j-2)}$$

General all-order construction technical and not so illuminating.

Ex. 4 construct $L^{(0)}_{1,0}$ $\sim$ needed for 2-pt closed-string disk amp

- start with the 'bosonic' products

$$l_1 \equiv Q_c \;,\quad l_2 \,,\quad \mathcal{L}^{(-1)}_{0,0} \,,\quad \mathcal{L}^{(-1)}_{1,0} \,,\quad m_{1,0} \,,\quad \mathcal{L}^{(-1)}_{0,1}$$

satisfying the homotopy relation

$$[Q_c, \mathcal{L}^{(-1)}_{1,0}] + [l_2, \mathcal{L}^{(-1)}_{0,0}] - \mathcal{L}^{(-1)}_{0,1}\, m_{1,0} = 0 \qquad (1)$$

- cyclic degree-odd gauge product

'bosonic', picture-deficit 2

$$\Lambda^{(0)}_{1,0} : \qquad [\eta_0^+, \Lambda^{(0)}_{1,0}] = -2\, \mathcal{L}^{(-1)}_{1,0}$$

e.g. $\Lambda^{(0)}_{1,0} = -\xi_0^+ \mathcal{L}^{(-1)}_{1,0} - \mathcal{L}^{(-1)}_{1,0}\, \xi_0^+$

- cyclic degree-even picture-deficit 1 product

$$\mathcal{L}^{(0)}_{1,0} = -[Q_c, \Lambda^{(0)}_{1,0}] - [\underbrace{\lambda_2^{(1,0)} + \bar{\lambda}_2^{(0,1)}}_{=2\xi_0^+ \circ \lambda_2}, \mathcal{L}^{(-1)}_{0,0}]$$

$$- \underbrace{\Lambda^{(0)}_{0,1}}_{= -\xi_0^+ \mathcal{L}^{(-1)}_{0,1}} m_{1,0} \qquad + \mathcal{L}^{(-1)}_{0,1} \underbrace{\mu^{(1)}_{1,0}}_{= -m_{1,0}\,\xi_0^+}$$

$$(1) \Rightarrow [\eta_0^+, \mathcal{L}^{(0)}_{1,0}] = 0$$

homotopy relation:

$$0 = [Q_c, \mathcal{L}^{(0)}_{1,0}] + [\underbrace{L_2^{(1,0)} + L_2^{(0,1)}}_{=[\ell_1, \lambda_2^{(1,0)} + \bar{\lambda}_2^{(0,1)}]}, \mathcal{L}^{(-1)}_{0,0}]$$

$$- \underbrace{\mathcal{L}^{(0)}_{0,1}}_{= -Q_c \Lambda^{(0)}_{0,1} - \Lambda^{(0)}_{0,1} Q_0} m_{1,0} \qquad - \mathcal{L}^{(-1)}_{0,1} \underbrace{M^{(1)}_{1,0}}_{= Q_0 \mu^{(1)}_{1,0} - \mu^{(1)}_{1,0} Q_c} \qquad (2)$$

- cyclic degree-odd gauge product

$$\Lambda^{(1)}_{1,0} : \qquad [\eta_0^+, \Lambda^{(1)}_{1,0}] = -\mathcal{L}^{(0)}_{1,0}$$

e.g. $\Lambda^{(1)}_{1,0} = -\frac{1}{2}\left(\xi_0^+ \mathcal{L}^{(0)}_{1,0} + \mathcal{L}^{(0)}_{1,0}\xi_0^+\right)$

- cyclic degree-even picture-deficit 0 product

$$\mathcal{L}^{(1)}_{1,0} = \frac{1}{2}\Big(-[Q_c, \Lambda^{(1)}_{1,0}] - [\underbrace{\lambda_2^{(1,1)} + \bar{\lambda}_2^{(1,1)}}_{= \xi_0 \circ L_2^{(0,1)} + \bar{\xi}_0 \circ L_2^{(1,0)}}, \mathcal{L}^{(-1)}_{0,0}]$$

$$- \Lambda^{(0)}_{0,1} M^{(1)}_{1,0} + \mathcal{L}^{(0)}_{0,1} \mu^{(1)}_{1,0} \Big)$$

$(2) \Rightarrow [\eta_0^+, \mathcal{L}^{(1)}_{1,0}] = 0$

homotopy relation:

$$0 = [Q_c, \mathcal{L}_{1,0}^{(1)}] + [\underbrace{L_2^{(1,1)}}_{= \frac{1}{2}[Q_c,\, \lambda_2^{(1,1)} + \bar{\lambda}_2^{(1,1)}]}, \mathcal{L}_{0,0}^{(-1)}] - \mathcal{L}_{0,1}^{(0)} M_{1,0}^{(1)}$$

- apply $[\eta_0^-, \cdot\,]$ to obtain degree-odd superstring products

$$L_{1,0}^{(0)} = [\eta_0^-, \mathcal{L}_{1,0}^{(1)}]$$

naive application of Munich construction in $\mathcal{H}_c^s$

$$= \frac{1}{2}\Big([Q_c, \lambda_{1,0}^{(0)}] - [\lambda_2^{(1,1)} + \bar{\lambda}_2^{(1,1)}, L_{0,0}^{(-2)}] - \underbrace{\lambda_{0,1}^{(-1)}}_{= [\eta_0^-, \Lambda_{0,1}^{(0)}]} M_{1,0}^{(1)} + \underbrace{L_{0,1}^{(-1)}}_{= [\eta_0^-, \mathcal{L}_{0,1}^{(0)}]} \mu_{1,0}^{(1)} \Big)$$

$$+ \frac{1}{4}[L_2^{(1,0)} - L_2^{(0,1)}, \mathcal{L}_{0,0}^{(-1)}]$$

correction to ensure that $[\eta_0^-, L_{1,0}^{(0)}] = 0$

Correct superstring homotopy relation:

$$0 = [Q_c, L^{(0)}_{1,0}] + [L^{(1,1)}_2, L^{(-2)}_{0,0}] + L^{(-1)}_{0,1} M^{(1)}_{1,0}$$

$\square$

Application: compute observables for perturbative O-C solns.

Marginal deformation

$$\Phi^*(\mu) = \mu V - \frac{1}{2}\mu^2 \frac{b_0^+}{L_0^+} \bar{P}_0^+ L_2^{(1,1)}(V,V) + O(\mu^3)$$

$$\Psi^*(\mu) = -\mu \frac{b_0}{L_0} \bar{P}_0 M_{1,0}^{(1)}(V) + O(\mu^2)$$

assume:

- $0 = QV = P_0^+ L_2^{(1,1)}(V,V)$ ⇝ bulk exact marg.
- $0 = P_0 M_{1,0}^{(1)}(V)$ ⇝ bdy exact marg.

[e.g.:

- Narain lattice: $V = -\frac{1}{2}\varepsilon_{ij}\, c\bar{c}\, \psi^i \bar{\psi}^j e^{-\phi-\bar{\phi}}$ + Dp-brane
- ghost dilaton

  ⋮]

0-, 1-, 2-pt amps
at zero momentum

Shifted disk partition function

$$
\begin{aligned}
\hat{Z}_D = Z_D &+ \mu\, \omega_c^s\left(V, \tilde{L}_{0,0}^{(-2)}\right) \\
&+ \frac{1}{2}\mu^2 \omega_c^s\left(V, \tilde{L}_{1,0}^{(0)}(V)\right) \\
&+ O(\mu^3)
\end{aligned}
$$

with

$$
\tilde{L}_{0,0}^{(-2)} = P_0^+ L_{0,0}^{(-2)}
$$

$$
\tilde{L}_{1,0}^{(0)} = P_0^+ L_{1,0}^{(0)} - L_2^{(1,1)} \frac{b_0^+}{L_0^+} \bar{P}_0^+ L_{0,0}^{(-2)} - L_{0,1}^{(-1)} \frac{b_0}{L_0} \bar{P}_0 M_{1,0}^{(1)}
$$

Expand in terms of 'bosonic' products and explicit X_0, ξ_0 contours:

$$\hat{Z}_D = Z_D + \mu\, \omega_c^s\left(V, \tilde{\ell}_{0,0}\right)$$

$P_0^+ \ell_{1,0} - \ell_2 \frac{b_0^+}{L_0^+} \bar{P}_0^+ \ell_{0,0} - \ell_{0,1} \frac{b_0}{L_0} \bar{P}_0 m_{1,0}$

$\Rightarrow$ S-matrix elt

$$+ \frac{1}{2}\mu^2 \Big[\omega_c^s\left(X_0 V, \tilde{\ell}_{1,0}(\bar{X}_0 V)\right)$$

+ terms localized at open and closed degeneration $\Big]$

involving P_0^+, P_0, $X_0^\pm, \xi_0^\pm$

$$+ O(\mu^3)$$

Can explicitly evaluate [see Carlo's talk]

$$\frac{1}{2}\omega_c^s\left(X_0 V, \tilde{\ell}_{1,0}(\bar{X}_0 V)\right) =$$

$$= \frac{1}{8\pi^2}\left(\int_0^{1/\lambda_0^2} \frac{ds}{s}\, s^{\varepsilon_0} + \int_{1/\lambda_0^2}^{1} \frac{ds}{s}\, (y(s))^{\varepsilon_c} \right) \times$$

$$\times \left\langle X_0 V(i,-i)\, b_0\, \bar{X}_0\, V(is,-is) \right\rangle \Big|_{\substack{\varepsilon_0 \to 0 \\ \varepsilon_c \to 0}}$$

(actually localizes at degenerations when $N=2$ ws SUSY present

For Narain-lattice deformations: $\rightsquigarrow V = -\frac{1}{2}\varepsilon_{ij}\, c\bar{c}\, \psi^i \bar{\psi}^j e^{-\phi-\bar{\phi}}$

- $\frac{1}{2} w_c^s\left(X_0 V, \tilde{\ell}_{1,0}(\bar{X}_0 V)\right) = \frac{1}{32\pi^2}\, \mathrm{tr}\left[\tilde{\varepsilon}\, g^{-1} \tilde{\varepsilon}\, g^{-1}\right]$

$$\tilde{\varepsilon} = \varepsilon\, \Omega^T$$

- localized contributions $= -\frac{1}{64\pi^2}\left(\mathrm{tr}\left[g^{-1}\tilde{\varepsilon}\right]^2 + \mathrm{tr}\left[g^{-1}\tilde{\varepsilon}\, g^{-1}\tilde{\varepsilon}^T\right]\right)$

Total shift in the disk partition function:

$$\hat{Z}_D = Z_D - \frac{1}{2\pi^2 g_s}\left(\frac{1}{4}\mathrm{tr}\left[\tilde{\varepsilon}\, g^{-1}\right] + \frac{1}{32}\mathrm{tr}\left[g^{-1}\tilde{\varepsilon}\right]^2 + \frac{1}{32}\mathrm{tr}\left[g^{-1}\tilde{\varepsilon}\, g^{-1}\tilde{\varepsilon}^T\right] - \frac{1}{16}\mathrm{tr}\left[\tilde{\varepsilon}\, g^{-1}\tilde{\varepsilon}\, g^{-1}\right]\right)$$

$+ O(\varepsilon^3)$

$$\varepsilon = \varepsilon_6 + \frac{1}{2}\varepsilon_6 g^{-1}\varepsilon_6 + O(\varepsilon_6^3)$$

(have to redo using NSNS super-SFT)

$\Rightarrow$ identical results as in the bosonic setting

SUMMARY & OUTLOOK

$$(\mathrm{CFT}, g_s, \|B\rangle\rangle) \xrightarrow[\Phi^*,\ \Psi^*]{} (\widehat{\mathrm{CFT}}, \hat{g}_s, \|\hat{B}\rangle\rangle)$$

↳ can provide gauge-inv. observables for classical O-C solns using on-shell amps for fluctuations around Φ^*, Ψ^*

↳ can compute all off-shell superstring disk amps

given the 'bosonic' products

- Munich construction for $b > 1$ (or even $g > 0$) ?
- D-brane deformations in non-trivial backgrounds ?
- RR-defs ?

blowing up orbifold singularities, Gepner points, ...